

WALSH'S ARITHMETIC FOR UPPER GRADES

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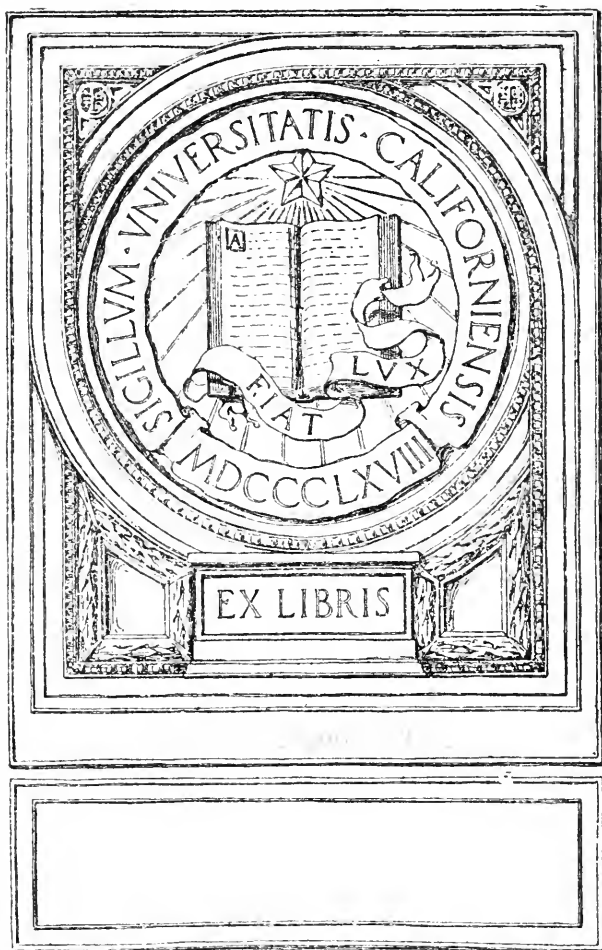
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AN ALGEBRA

ARITHMETIC

FOR UPPER GRADES

BY

JOHN H. WALSH

ASSOCIATE CITY SUPERINTENDENT OF SCHOOLS
THE CITY OF NEW YORK



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PREFACE.

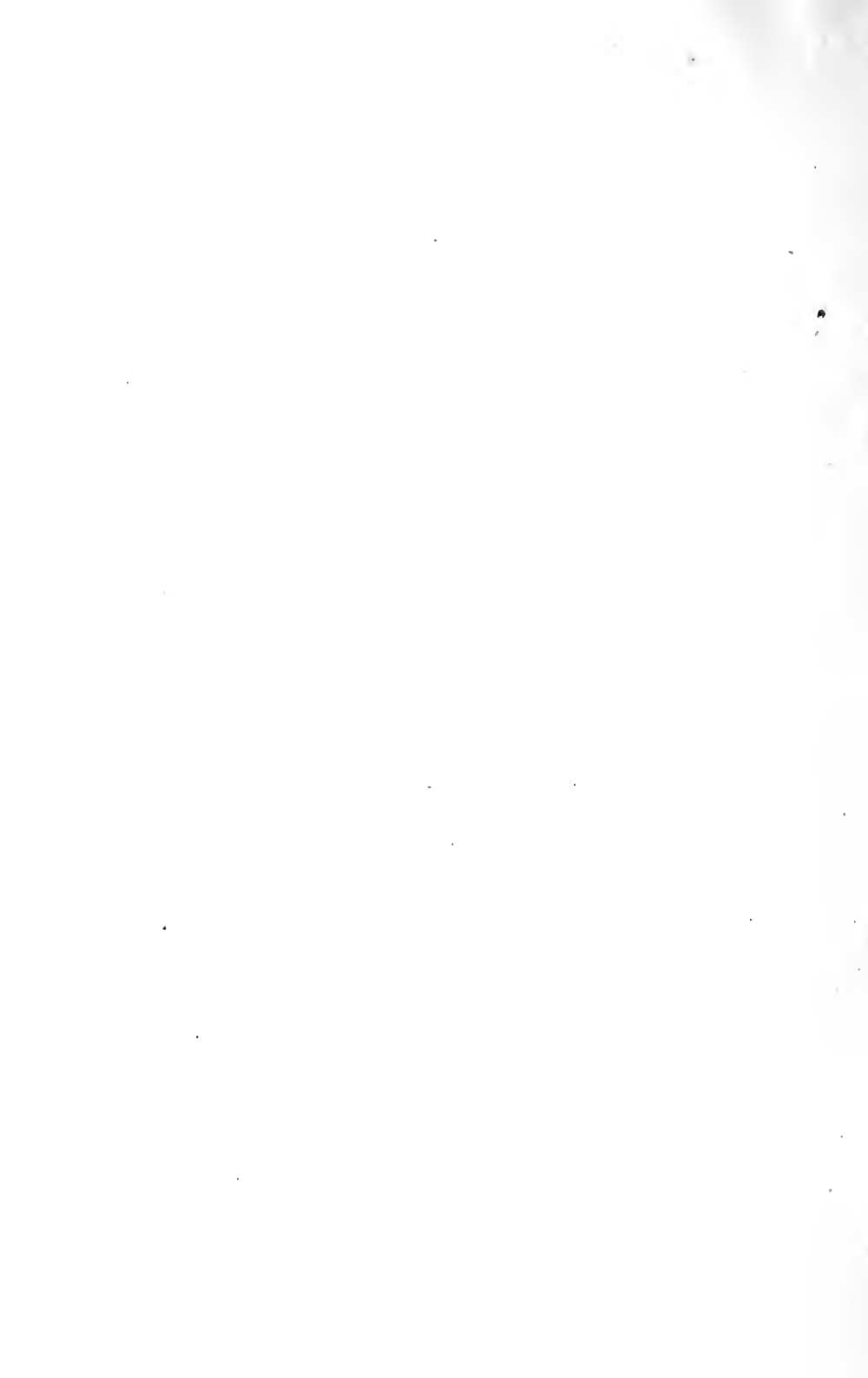
AN Arithmetic for Upper Grades, while intended chiefly for pupils of the last two years of the elementary school, has been arranged to include the work of the sixth grade.

The seventh year portion comprises a simple, but systematic, treatment of commercial arithmetic, including percentage with its several applications, and elementary exercises involving the employment of common business forms. This is preceded by reviews of fractions and decimals, and is followed by miscellaneous problems, oral and written.

The eighth year section contains the remaining topics of the ordinary course, prefaced by a review of compound denominate numbers and simple measurements. The miscellaneous problems that immediately follow are limited to the subjects treated in this portion, so as to be available for use in such schools as teach the seventh and eighth grades in combined classes alternating the work of each year.

A scientific treatment of numbers and processes is next presented, which may be taken up at any stage. The miscellaneous reviews that follow cover all the ground previously studied.

In many schools whose courses of study require an advanced text-book in the sixth year, it is customary to begin the arithmetic work of this grade with the development of formal definitions, principles, rules, etc. In this case, the section on numbers and processes should first be taken up, then the fraction and decimal reviews, followed by the reviews of compound numbers and measurements.



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AN ARITHMETIC FOR UPPER GRADES.

CHAPTER I.

REVIEW OF FRACTIONS.

1. As a preliminary to the regular percentage work of the seventh year, it will often be found profitable to give a short time to the review of fractions, common and decimal. At intervals throughout the year, a few minutes of an arithmetic period should be spent in rapid oral reviews, employing the drills and the sight exercises of this chapter.

Drill Exercises.

NOTE. The fractions in the answers should be given in their lowest terms.

2. Add:

- | | | | | | | | | | | | |
|-----|----------------|-----|----------------|-----|----------------|-----|----------------|-----|----------------|-----|-----------------|
| 1. | $\frac{1}{3}$ | 2. | $\frac{2}{5}$ | 3. | $\frac{4}{9}$ | 4. | $\frac{2}{5}$ | 5. | $\frac{2}{9}$ | 6. | $\frac{4}{9}$ |
| | $\frac{1}{3}$ | | $\frac{1}{5}$ | | $\frac{1}{9}$ | | $\frac{2}{5}$ | | $\frac{2}{9}$ | | $\frac{4}{9}$ |
| | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> |
| 7. | $\frac{2}{3}$ | 8. | $\frac{3}{5}$ | 9. | $\frac{5}{9}$ | 10. | $\frac{2}{3}$ | 11. | $\frac{4}{5}$ | 12. | $\frac{8}{9}$ |
| | $\frac{1}{3}$ | | $\frac{2}{5}$ | | $\frac{4}{9}$ | | $\frac{2}{3}$ | | $\frac{3}{5}$ | | $\frac{5}{9}$ |
| | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> |
| 13. | $3\frac{1}{3}$ | 14. | $4\frac{2}{5}$ | 15. | $6\frac{4}{9}$ | 16. | $3\frac{2}{5}$ | 17. | $6\frac{2}{9}$ | 18. | $8\frac{4}{9}$ |
| | $\frac{1}{3}$ | | $\frac{1}{5}$ | | $\frac{1}{9}$ | | $\frac{2}{5}$ | | $\frac{2}{9}$ | | $\frac{4}{9}$ |
| | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> |
| 19. | $5\frac{2}{3}$ | 20. | $6\frac{3}{5}$ | 21. | $9\frac{5}{9}$ | 22. | $6\frac{2}{3}$ | 23. | $6\frac{4}{5}$ | 24. | $13\frac{8}{9}$ |
| | $\frac{1}{3}$ | | $\frac{2}{5}$ | | $\frac{4}{9}$ | | $\frac{2}{3}$ | | $\frac{3}{5}$ | | $\frac{5}{9}$ |
| | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> | | <hr/> |

Add:

- | | | | | | | | | | | | |
|-----|---|-----|--|-----|--|-----|--|-----|---|-----|---|
| 25. | $4\frac{1}{3}$
<u>$3\frac{1}{3}$</u> | 26. | $5\frac{2}{5}$
<u>$6\frac{1}{5}$</u> | 27. | $9\frac{4}{9}$
<u>$2\frac{1}{9}$</u> | 28. | $5\frac{2}{5}$
<u>$6\frac{2}{5}$</u> | 29. | $8\frac{2}{9}$
<u>$7\frac{2}{9}$</u> | 30. | $6\frac{4}{9}$
<u>$9\frac{4}{9}$</u> |
| 31. | $4\frac{2}{3}$
<u>$3\frac{1}{3}$</u> | 32. | $6\frac{3}{5}$
<u>$3\frac{2}{5}$</u> | 33. | $8\frac{5}{9}$
<u>$4\frac{4}{9}$</u> | 34. | $9\frac{2}{3}$
<u>$8\frac{2}{3}$</u> | 35. | $7\frac{4}{5}$
<u>$6\frac{3}{5}$</u> | 36. | $2\frac{8}{9}$
<u>$3\frac{5}{9}$</u> |
| 37. | $\frac{1}{4}$
<u>$\frac{1}{4}$</u> | 38. | $4\frac{1}{6}$
<u>$\frac{1}{6}$</u> | 39. | $\frac{5}{8}$
<u>$3\frac{1}{8}$</u> | 40. | $5\frac{7}{12}$
<u>$9\frac{1}{12}$</u> | 41. | $\frac{3}{8}$
<u>$\frac{3}{8}$</u> | 42. | $5\frac{4}{9}$
<u>$\frac{2}{9}$</u> |
| 43. | $\frac{3}{10}$
<u>$8\frac{3}{10}$</u> | 44. | $9\frac{5}{12}$
<u>$3\frac{5}{12}$</u> | 45. | $\frac{7}{8}$
<u>$\frac{5}{8}$</u> | 46. | $3\frac{8}{9}$
<u>$\frac{7}{9}$</u> | 47. | $\frac{7}{10}$
<u>$9\frac{9}{10}$</u> | 48. | $3\frac{5}{12}$
<u>$7\frac{11}{12}$</u> |

3. Subtract:

- | | | | | | | | | | | | |
|-----|--|-----|--|-----|--|-----|---|-----|--|-----|--|
| 1. | $\frac{4}{5}$
<u>$\frac{1}{5}$</u> | 2. | $\frac{7}{9}$
<u>$\frac{5}{9}$</u> | 3. | $6\frac{3}{5}$
<u>$\frac{2}{5}$</u> | 4. | $8\frac{8}{9}$
<u>$\frac{4}{9}$</u> | 5. | $7\frac{4}{5}$
<u>$2\frac{2}{5}$</u> | 6. | $6\frac{5}{9}$
<u>$4\frac{4}{9}$</u> |
| 7. | 8
<u>$\frac{1}{3}$</u> | 8. | 6
<u>$\frac{2}{5}$</u> | 9. | 7
<u>$\frac{5}{9}$</u> | 10. | 4
<u>$3\frac{2}{3}$</u> | 11. | 6
<u>$5\frac{2}{5}$</u> | 12. | 8
<u>$7\frac{5}{9}$</u> |
| 13. | 8
<u>$1\frac{1}{3}$</u> | 14. | 6
<u>$1\frac{2}{5}$</u> | 15. | 7
<u>$1\frac{5}{9}$</u> | 16. | 9
<u>$3\frac{2}{3}$</u> | 17. | 6
<u>$4\frac{2}{5}$</u> | 18. | 8
<u>$2\frac{5}{9}$</u> |
| 19. | $8\frac{1}{3}$
<u>$1\frac{2}{3}$</u> | 20. | $6\frac{1}{5}$
<u>$1\frac{2}{5}$</u> | 21. | $7\frac{1}{9}$
<u>$1\frac{5}{9}$</u> | 22. | $9\frac{1}{3}$
<u>$3\frac{2}{3}$</u> | 23. | $6\frac{2}{5}$
<u>$4\frac{3}{5}$</u> | 24. | $8\frac{4}{9}$
<u>$2\frac{5}{9}$</u> |
| 25. | $\frac{5}{6}$
<u>$\frac{1}{6}$</u> | 26. | $\frac{7}{8}$
<u>$\frac{5}{8}$</u> | 27. | $\frac{7}{9}$
<u>$\frac{4}{9}$</u> | 28. | $1\frac{1}{12}$
<u>$\frac{5}{12}$</u> | 29. | $9\frac{5}{8}$
<u>$\frac{1}{8}$</u> | 30. | $8\frac{8}{9}$
<u>$\frac{2}{9}$</u> |

4. Multiply:

- | | | | | | | | |
|----|------------------------|-----|-------------------------|-----|---------------------|-----|--------------------------|
| 1. | $\frac{1}{3} \times 2$ | 2. | $3 \times \frac{1}{4}$ | 3. | $\frac{1}{5}$ of 4 | 4. | $5 \times \frac{1}{6}$ |
| 5. | $\frac{1}{7} \times 9$ | 6. | $17 \times \frac{1}{8}$ | 7. | $\frac{1}{9}$ of 23 | 8. | $31 \times \frac{1}{10}$ |
| 9. | $\frac{2}{5} \times 2$ | 10. | $3 \times \frac{2}{7}$ | 11. | $\frac{5}{6}$ of 5 | 12. | $7 \times \frac{5}{8}$ |

Multiply :

$$\begin{array}{llll} 13. \quad \frac{3\frac{1}{3}}{2} & 14. \quad \frac{5\frac{1}{4}}{3} & 15. \quad \frac{7\frac{1}{5}}{4} & 16. \quad \frac{9\frac{1}{6}}{5} & 17. \quad \frac{8\frac{1}{7}}{6} \end{array}$$

$$\begin{array}{llll} 18. \quad \frac{2\frac{2}{5}}{2} & 19. \quad \frac{5\frac{2}{7}}{3} & 20. \quad \frac{7\frac{5}{8}}{5} & 21. \quad \frac{3\frac{2}{9}}{8} & 22. \quad \frac{9\frac{1}{6}}{7} \end{array}$$

$$23. \quad \frac{1}{4} \text{ of } 2 \quad 24. \quad 3 \times \frac{1}{6} \quad 25. \quad \frac{1}{8} \times 4 \quad 26. \quad 6 \times \frac{1}{9}$$

$$27. \quad \frac{1}{4} \text{ of } 4 \quad 28. \quad 6 \times \frac{1}{6} \quad 29. \quad \frac{1}{8} \times 16 \quad 30. \quad 27 \times \frac{1}{9}$$

$$31. \quad \frac{1}{4} \text{ of } 6 \quad 32. \quad 9 \times \frac{1}{6} \quad 33. \quad \frac{1}{8} \times 20 \quad 34. \quad 33 \times \frac{1}{9}$$

$$35. \quad \frac{3}{8} \text{ of } 2 \quad 36. \quad 3 \times \frac{2}{9} \quad 37. \quad \frac{5}{6} \times 3 \quad 38. \quad 7 \times \frac{4}{7}$$

$$39. \quad \frac{2}{3} \text{ of } 15 \quad 40. \quad 24 \times \frac{3}{4} \quad 41. \quad \frac{5}{8} \times 12 \quad 42. \quad 3 \times \frac{7}{9}$$

$$\begin{array}{llll} 43. \quad \frac{4\frac{1}{4}}{2} & 44. \quad \frac{5\frac{1}{6}}{4} & 45. \quad \frac{6\frac{3}{8}}{2} & 46. \quad \frac{7\frac{2}{9}}{3} & 47. \quad \frac{8\frac{1}{10}}{6} \end{array}$$

$$\begin{array}{llll} 48. \quad \frac{6\frac{1}{4}}{6} & 49. \quad \frac{8\frac{1}{6}}{9} & 50. \quad \frac{3\frac{2}{3}}{3} & 51. \quad \frac{4\frac{3}{4}}{4} & 52. \quad \frac{6\frac{2}{5}}{5} \end{array}$$

5. Divide:

$$1. \quad \underline{2)4 \text{ fifths}} \quad 2. \quad \underline{3)6 \text{ sevenths}} \quad 3. \quad \underline{4)8 \text{ ninths}}$$

NOTE. In dividing $\frac{4}{5}$ by 2, the pupil may think 2 into 4 fifths, or $\frac{1}{2}$ of 4 fifths, or 4 fifths divided by 2. These and the following examples are placed in the short-division form to lead pupils to refrain, at times, in written work from changing the mixed number in the dividend to an improper fraction when the divisor is a whole number.

$$4. \quad \underline{7)\frac{14}{5}} \quad 5. \quad \underline{8)\frac{16}{5}} \quad 6. \quad \underline{5)\frac{25}{6}} \quad 7. \quad \underline{6)\frac{12}{3}}$$

$$8. \quad \underline{2)\frac{4}{5}} \quad 9. \quad \underline{3)\frac{6}{7}} \quad 10. \quad \underline{4)\frac{8}{6}} \quad 11. \quad \underline{5)\frac{5}{5}}$$

$$12. \quad \underline{2)\frac{1}{2}} \quad 13. \quad \underline{3)\frac{1}{3}} \quad 14. \quad \underline{4)\frac{1}{2}} \quad 15. \quad \underline{5)\frac{1}{3}}$$

$$16. \quad \underline{2)\frac{12}{2}} \quad 17. \quad \underline{3)\frac{9}{3}} \quad 18. \quad \underline{4)\frac{16}{2}} \quad 19. \quad \underline{5)\frac{15}{3}}$$

Divide :

20. $6 \overline{)18\frac{1}{2}}$

21. $7 \overline{)21\frac{1}{5}}$

22. $8 \overline{)40\frac{1}{2}}$

23. $9 \overline{)63\frac{1}{3}}$

24. $5 \overline{)\frac{5}{2}}$

25. $5 \overline{)2\frac{1}{2}}$

26. $2 \overline{)\frac{4}{3}}$

27. $2 \overline{)1\frac{1}{3}}$

28. $4 \overline{)2\frac{2}{3}}$

29. $5 \overline{)3\frac{3}{4}}$

30. $6 \overline{)3\frac{3}{5}}$

31. $7 \overline{)4\frac{2}{3}}$

32. $2 \overline{)19\frac{1}{3}}$

33. $4 \overline{)26\frac{2}{3}}$

34. $5 \overline{)28\frac{3}{4}}$

35. $6 \overline{)33\frac{3}{5}}$

36. $2 \overline{)\frac{3}{2}}$

37. $2 \overline{)1\frac{1}{2}}$

38. $3 \overline{)\frac{4}{3}}$

39. $3 \overline{)1\frac{1}{3}}$

40. $2 \overline{)17\frac{1}{2}}$

41. $3 \overline{)16\frac{1}{3}}$

42. $4 \overline{)19\frac{1}{3}}$

43. $5 \overline{)42\frac{2}{3}}$

44. $6 \overline{)43\frac{1}{4}}$

45. $7 \overline{)40\frac{1}{2}}$

46. $8 \overline{)41\frac{1}{3}}$

47. $9 \overline{)46\frac{1}{5}}$

6. Preliminary Exercises.

1. How many baseballs at $\$ \frac{1}{2}$ each can be purchased for $\$ \frac{1}{2}$? For $\$ 1$? For $\$ 1\frac{1}{2}$?

2. $\frac{1}{2} \overline{)\frac{1}{2}}$

3. $\frac{1}{2} \overline{)1}$

4. $\frac{1}{2} \overline{)1\frac{1}{2}}$

5. $\frac{1}{2} \overline{)2}$

6. $\frac{1}{2} \overline{)2\frac{1}{2}}$

7. $\frac{1}{2} \overline{)8}$

8. $\frac{1}{2} \overline{)20}$

9. $\frac{1}{3} \overline{)6}$

10. $\frac{1}{3} \overline{)12}$

11. $\frac{1}{4} \overline{)12}$

How is the quotient obtained in each case?

12. $\frac{3}{2} \overline{)\frac{3}{2}}$

13. $1\frac{1}{2} \overline{)1\frac{1}{2}}$

14. $\frac{3}{2} \overline{)\frac{6}{2}}$

15. $1\frac{1}{2} \overline{)3}$

16. $1\frac{1}{2} \overline{)6}$

Multiply the divisor and the dividend in each of the five preceding examples by 2 :

17. $3 \overline{)3}$

18. $3 \overline{)3}$

19. $3 \overline{)6}$

20. $3 \overline{)6}$

21. $3 \overline{)12}$

How do the quotients compare in each case?

7. 1. Divide 12 by $1\frac{1}{2}$.

2. Divide 21 by $\frac{3}{4}$.

$$\begin{array}{r} 1\frac{1}{2} \overline{)12} \\ \times 2 \quad \times 2 \\ \hline 3 \overline{)24} \\ 8 \text{ Ans.} \end{array}$$

$$\begin{array}{r} \frac{3}{4} \overline{)21} \\ \times 4 \quad \times 4 \\ \hline 3 \overline{)84} \\ 28 \text{ Ans.} \end{array}$$

PROOF. $1\frac{1}{2} \times 8 = 12.$

PROOF. $28 \times \frac{3}{4} = 21.$

NOTE. In mental work it is often convenient to change a fractional divisor to a whole number by multiplying the divisor by the denominator of the fraction, the dividend being multiplied by the same number. Divide the new dividend by the new divisor.

8. Drill Exercises.

Divide:

- | | | | | |
|----------------------|-----------------------|----------------------|----------------------------------|--------------------------------|
| 1. $1\frac{1}{2})9$ | 6. $2\frac{1}{2})15$ | 11. $\frac{3}{4})12$ | 16. $1\frac{1}{2})7\frac{1}{2}$ | 21. $\frac{3}{4})2\frac{1}{4}$ |
| 2. $1\frac{1}{2})15$ | 7. $3\frac{1}{2})21$ | 12. $\frac{2}{3})12$ | 17. $1\frac{1}{4})8\frac{3}{4}$ | 22. $\frac{2}{3})2\frac{2}{3}$ |
| 3. $1\frac{1}{2})18$ | 8. $2\frac{1}{4})27$ | 13. $\frac{4}{5})12$ | 18. $1\frac{1}{3})9\frac{1}{3}$ | 23. $\frac{4}{5})4\frac{4}{5}$ |
| 4. $1\frac{1}{4})15$ | 9. $3\frac{1}{4})26$ | 14. $\frac{5}{6})10$ | 19. $1\frac{1}{4})7\frac{1}{2}$ | 24. $\frac{5}{6})5\frac{5}{6}$ |
| 5. $1\frac{1}{4})20$ | 10. $1\frac{2}{3})15$ | 15. $\frac{7}{8})14$ | 20. $1\frac{2}{3})16\frac{2}{3}$ | 25. $\frac{6}{7})6\frac{6}{7}$ |

9. Oral Problems.

1. A farmer sold $15\frac{1}{2}$ cords of wood in January and $10\frac{1}{2}$ cords in February. How many cords did he sell in all?

2. From a piece of cloth containing 30 yards, $12\frac{1}{2}$ yards are sold. How many yards remain?

3. A rectangular field is $12\frac{1}{4}$ rods long and $7\frac{1}{4}$ rods wide. How many rods of fence will be needed to inclose it?

4. How many $\frac{1}{2}$ -pound packages will $24\frac{1}{2}$ pounds of candy make?

5. A traveler walked $60\frac{3}{4}$ miles in 3 days. How many miles a day did he average?

6. How many square rods are there in a field $20\frac{1}{2}$ rods long and 10 rods wide?

7. Mr. Yates pays $\$17\frac{1}{4}$ for carpet and $\$20\frac{1}{4}$ for furniture. What is the amount of his bill?

8. How many minutes are there in $\frac{1}{6}$ of a day?

9. At 60 pounds per bushel, what will $\frac{3}{4}$ bushel weigh?

10. How many yards of cloth at $\$1\frac{1}{2}$ per yard can be bought for \$12?

10. Written Problems.

1. A boy sold $16\frac{3}{4}$ dozen eggs at one time and $20\frac{3}{4}$ dozen at another time. How many eggs did he sell?

2. Find the sum of four numbers, two of which are $15\frac{4}{5}$ and $19\frac{7}{15}$, respectively, the third being equal to the sum of these two, and the fourth being equal to their difference.

3. Two trains start from the same point and move in opposite directions, each at the rate of $32\frac{1}{4}$ miles per hour. How far apart are they in 4 hours?

4. What is the total weight of 16 barrels of sugar, averaging $310\frac{1}{4}$ pounds each?

5. A crop of wheat averaged $12\frac{1}{2}$ bushels per acre. How many acres were required to produce 500 bushels?

6. How many square rods are there in a rectangular field $160\frac{1}{3}$ rods by 84 rods?

7. A train starting at 10.45 A.M. reaches a town 140 miles distant at 2.15 P.M. How many miles per hour does it average?

8. If 3 eighths of a number is 147, what is 1 eighth of the number? What is the number?

9. A rectangular lot is 120 feet long. Its width is $\frac{9}{10}$ of its length. How many running feet of fence will be required to inclose it? (Make a diagram.)

10. How many gallons are there in $11\frac{1}{3}$ barrels of $31\frac{1}{2}$ gallons each?

11. Sight Exercises.

NOTE. To accustom the pupils to avoid unnecessary figures, frequent drills in sight and blackboard exercises are important. Pupils should give orally the answers to the following examples, or should promptly write the answer to each at a signal, the pupil being expected to know the answer before beginning to write.

Add :

$$\begin{array}{r} 1. \quad 24\frac{1}{2} \\ \quad 3\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 17\frac{2}{3} \\ \quad 5\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 48\frac{5}{8} \\ \quad 3\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 42\frac{5}{8} \\ \quad 8\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 8\frac{1}{2} \\ \quad 36\frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 84\frac{1\frac{3}{4}}{16} \\ \quad 9\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 93\frac{1}{4} \\ \quad 7\frac{3}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 3\frac{3}{4} \\ \quad 91\frac{1}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 46\frac{5}{9} \\ \quad 7\frac{1}{6} \\ \hline \end{array}$$

12. Blackboard Exercises.

NOTE. Pupils are expected to write only the answers to the following examples, but time should be allowed them to write the total of each column as they obtain it. These exercises are designed to show pupils that it is not always necessary to rewrite the fractions with a common denominator.

Add :

$$\begin{array}{r} 1. \quad 24\frac{1}{2} \\ \quad 6\frac{3}{4} \\ \quad 59\frac{1}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 40\frac{1}{3} \\ \quad 28\frac{1}{6} \\ \quad 5\frac{1}{9} \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 47\frac{1}{2} \\ \quad 7\frac{3}{4} \\ \quad 59\frac{1}{16} \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 47\frac{2}{3} \\ \quad 18\frac{1}{2} \\ \quad 8\frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 48\frac{1}{3} \\ \quad 32\frac{1}{6} \\ \quad 7\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 46\frac{1}{2} \\ \quad 8\frac{7}{8} \\ \quad 32\frac{1}{16} \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 84\frac{1}{2} \\ \quad 10\frac{2}{5} \\ \quad 3\frac{1}{10} \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 23\frac{1}{2} \\ \quad 45\frac{2}{5} \\ \quad 6\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 30\frac{2}{3} \\ \quad 41\frac{1}{4} \\ \quad 5\frac{1}{12} \\ \hline \end{array}$$

13. Sight Exercises.

Subtract :

$$\begin{array}{r} 1. \quad 18\frac{3}{4} \\ \quad 6\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 72\frac{1}{2} \\ \quad 2\frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 40\frac{7}{8} \\ \quad 4\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 54\frac{1}{2} \\ \quad 3\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 27\frac{1}{2} \\ \quad 5\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 80\frac{1}{4} \\ \quad 7\frac{1}{3} \\ \hline \end{array}$$

Subtract:

$$\begin{array}{r} 7. \quad 36\frac{1}{3} \\ \quad 9\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 62\frac{1}{10} \\ \quad 8\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 93\frac{1}{6} \\ \quad 10\frac{1}{3} \\ \hline \end{array}$$

14. Blackboard Exercises.

Subtract:

$$\begin{array}{r} 1. \quad 84\frac{1}{4} \\ \quad 29\frac{7}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 40\frac{1}{2} \\ \quad 16\frac{5}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 63\frac{1}{2} \\ \quad 27\frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 90\frac{1}{16} \\ \quad 26\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 60\frac{2}{3} \\ \quad 23\frac{5}{6} \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 52\frac{1}{16} \\ \quad 34\frac{7}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 78\frac{5}{12} \\ \quad 39\frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 45\frac{7}{10} \\ \quad 18\frac{3}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 93\frac{2}{3} \\ \quad 47\frac{4}{9} \\ \hline \end{array}$$

15. Sight Exercises.

Multiply:

$$\begin{array}{r} 1. \quad 12\frac{1}{2} \\ \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 16 \\ \quad 4\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 10\frac{1}{6} \\ \quad 12 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 20\frac{2}{3} \\ \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 24 \\ \quad 2\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 12\frac{3}{7} \\ \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 21\frac{3}{4} \\ \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 48 \\ \quad 1\frac{1}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 40\frac{5}{9} \\ \quad 9 \\ \hline \end{array}$$

16. Blackboard Exercises.

Multiply:

$$\begin{array}{r} 1. \quad 124\frac{1}{2} \\ \quad 7 \\ \hline \end{array}$$

$$\begin{array}{r} 4. \quad 304\frac{5}{6} \\ \quad 12 \\ \hline \end{array}$$

$$\begin{array}{r} 7. \quad 251\frac{2}{11} \\ \quad 10 \\ \hline \end{array}$$

$$\begin{array}{r} 2. \quad 320\frac{2}{3} \\ \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} 5. \quad 423\frac{4}{7} \\ \quad 2 \\ \hline \end{array}$$

$$\begin{array}{r} 8. \quad 222\frac{3}{10} \\ \quad 13 \\ \hline \end{array}$$

$$\begin{array}{r} 3. \quad 621\frac{3}{4} \\ \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 6. \quad 516\frac{5}{9} \\ \quad 5 \\ \hline \end{array}$$

$$\begin{array}{r} 9. \quad 201\frac{1}{9} \\ \quad 14 \\ \hline \end{array}$$

17. Sight Exercises.

Divide:

1. $2 \overline{)26\frac{1}{2}}$

4. $7 \overline{)78\frac{1}{6}}$

7. $6 \overline{)67\frac{1}{4}}$

2. $3 \overline{)39\frac{3}{4}}$

5. $8 \overline{)17\frac{3}{5}}$

8. $5 \overline{)51\frac{2}{3}}$

3. $4 \overline{)36\frac{4}{5}}$

6. $9 \overline{)36\frac{9}{10}}$

9. $4 \overline{)27\frac{3}{4}}$

18. Blackboard Exercises.

Divide:

1. $2 \overline{)246\frac{1}{3}}$

4. $5 \overline{)849\frac{7}{8}}$

7. $8 \overline{)649\frac{1}{7}}$

2. $3 \overline{)459\frac{1}{2}}$

5. $6 \overline{)273\frac{1}{2}}$

8. $9 \overline{)833\frac{4}{5}}$

3. $4 \overline{)723\frac{1}{5}}$

6. $7 \overline{)723\frac{2}{5}}$

9. $10 \overline{)537\frac{2}{3}}$

19. Written Exercises.

NOTE. Determine the common denominator by inspection.

Find results:

1. $8\frac{1}{2} + 7\frac{1}{4} + 13\frac{5}{6} + 42\frac{3}{8}$

3. $28\frac{3}{4} + 45\frac{4}{5} + 83\frac{3}{8} + 96\frac{1}{6}$

2. $26\frac{2}{3} + 30\frac{2}{3} + 47\frac{1}{6} + 56\frac{5}{9}$

4. $35\frac{3}{5} + 56\frac{7}{12} + 97\frac{7}{8} + 48\frac{1}{2}\frac{3}{4}$

5. $19\frac{1}{2}\frac{9}{4} + 12\frac{5}{8} + 24\frac{2}{3}\frac{9}{6} + 87\frac{2}{5}$

6. $910\frac{3}{8} - 316\frac{7}{12}$

9. $862\frac{2}{8} - 258\frac{5}{8}$

7. $862\frac{3}{8} - 258\frac{2}{3}$

10. $683\frac{7}{10} - 423\frac{1}{15}$

8. $200\frac{5}{12} - 103\frac{7}{15}$

11. $709\frac{5}{3} - 357\frac{5}{12}$

To multiply two mixed numbers, reduce them to improper fractions, multiply the numerators together and the denominators together, and reduce the resulting fraction, if possible.

12. $4\frac{2}{3} \times 4\frac{4}{5}$

15. $15\frac{3}{8} \times 10\frac{2}{3}$

13. $2\frac{8}{11} \times 6\frac{2}{5}$

16. $17\frac{1}{2} \times 5\frac{3}{7}$

14. $3\frac{4}{7} \times 5\frac{4}{9}$

17. $12\frac{5}{6} \times 7\frac{4}{9}$

To divide by a fraction, multiply by the divisor inverted.

18. $4\frac{2}{7} \div 4\frac{1}{9}$

23. $(3\frac{3}{8} + 5\frac{5}{6}) \times 4\frac{2}{3}$

19. $7\frac{5}{8} \div 3\frac{7}{12}$

24. $(7\frac{5}{12} - 5\frac{5}{9}) \div 4\frac{1}{3}$

20. $5\frac{5}{8} \div 3\frac{3}{4}$

25. $6\frac{2}{3} \times (17\frac{3}{4} + 9\frac{5}{6})$

21. $16\frac{1}{2} \div 33\frac{1}{3}$

26. $(66\frac{2}{3} - 36\frac{4}{5}) \div 2\frac{2}{3}$

22. $20\frac{4}{5} \div 41\frac{2}{5}$

27. $(3\frac{4}{7} \times 5\frac{4}{9}) \div (7\frac{5}{8} \div 3\frac{7}{12})$

TYPE PROBLEMS.

MULTIPLICATION OR DIVISION: ONE OPERATION.

20. Preliminary Exercises.

1. At 12 cents per yard, find the cost of 2 yards of dress goods. Of $2\frac{1}{2}$ yards. Of $\frac{1}{2}$ yard. Of $\frac{3}{4}$ yard.

To indicate the operation required in each case, the sign of multiplication is employed: $12\phi \times 2$, $12\phi \times 2\frac{1}{2}$, $12\phi \times \frac{1}{2}$, $12\phi \times \frac{3}{4}$.

2. Find the price per yard when 2 yards cost 24 cents. When $2\frac{1}{2}$ yards cost 30 cents. When $\frac{1}{2}$ yard costs 6 cents. When $\frac{3}{4}$ yard costs 9 cents.

In each of these examples the price per yard is obtained by dividing the total cost by the number of yards: $24\phi \div 2$, $30\phi \div 2\frac{1}{2}$, $6\phi \div \frac{1}{2}$, $9\phi \div \frac{3}{4}$.

21. Oral Problems.

NOTE. In solving each of the following problems, the pupils should first state whether it is an example in multiplication or in division. They may easily determine this by mentally substituting a whole number for the fraction.

1. A 24-acre field is divided into plots of $\frac{3}{4}$ acre each. How many plots are there?

2. At $\$ \frac{7}{8}$ per bushel, find the cost of 56 bushels of wheat.

3. How many cords of wood in 32 piles containing $\frac{7}{8}$ cord each?

4. If a train goes $\frac{5}{6}$ mile in a minute, how many minutes will it take to go 60 miles?

5. A dealer's profit is $\frac{1}{4}$ of the cost. What is the cost, if his profit is \$24?

6. How many $\frac{3}{4}$ -pound packages can be filled from a 36-pound box of tea?

7. A drover sells $\frac{4}{5}$ of his herd of 120 cattle. How many does he sell?

8. Nine tenths of the pupils of a certain class are present. There are 27 present. How many pupils belong to the class?

9. If a man can do two fifths of a piece of work in a day, how long will it take him to do the whole work?

Number of days = 1 work \div $\frac{2}{5}$ work = $\frac{5}{2}$ work \div $\frac{2}{5}$ work = $5 \div 2$.

10. How long will it take a pipe discharging $\frac{3}{4}$ gallon per second to empty a tank containing 60 gallons?

22. Written Problems.

NOTE. Before solving the following problems, the required operation should be indicated in each case by the use of the proper sign.

1. Into how many building sites of $\frac{3}{4}$ acre each can a farm of 192 acres be divided?

Number of sites = $192 \text{ A.} \div \frac{3}{4} \text{ A.}$

2. Find the cost of 784 bushels of wheat at $\$1\frac{5}{8}$ per bushel.

Cost = $\$1\frac{5}{8} \times 784$.

3. How many loads, each containing $\frac{7}{8}$ cord, are there in 336 cords of wood?

4. What time will it take a train to go 195 miles at the rate of $\frac{4}{5}$ mile a minute?

5. At 95¢ per bushel, how many bushels of wheat can be bought for \$142.50?

6. How many bushels of wheat at $\$ \frac{1}{2} \frac{9}{10}$ per bushel can be bought for $\$ 142 \frac{1}{2}$?

7. If it takes $\frac{3}{4}$ yard of material to make an apron, how many yards will be required to make 144 aprons?

8. How many vests can be made from 144 yards of cloth, if $\frac{3}{4}$ yard is needed for each?

9. If three men working together can do $\frac{1}{20} + \frac{1}{25} + \frac{1}{30}$ of a piece of work in a day, how long will they require to do the whole work?

10. Find the cost of $\frac{1}{16}$ acre of land at $\$ 256$ per acre.

11. If a horse eats $\frac{2}{5}$ bale of hay in a week, how long will a bale last? 32 bales?

12. A farmer sold his farm for $\frac{4}{5}$ of its cost, which was $\$ 4800$. What did he receive for it?

13. A can do $\frac{7}{8}$ as much work in a day as B. How many days would he require to do a piece of work that B could finish in 105 days?

14. A and B together can do $\frac{1}{8}$ as much work as B alone. How many days would both working together require to do a piece of work which B can do in 105 days?

15. A dealer's profits average $\frac{1}{100}$ of the cost of the goods sold. How much does he gain on goods costing $\$ 275$?

16. If the weight of roasted coffee is $\frac{1}{20}$ of the weight of unroasted coffee, how many pounds of the latter will be required to make 221 pounds of roasted coffee?

SUGGESTION. In this problem and in the remaining four, the pupil may use x as follows:

Let x represent the number of pounds of unroasted coffee.

Then,
$$\frac{1}{20}x = 221$$
$$x = 221 \div \frac{1}{20}, \text{ etc.}$$

17. A certain number multiplied by $\frac{3}{4}$ gives $115 \frac{1}{2}$ as the result. What is the number?

18. What must be the capacity of a bin in cubic feet to hold 385 bushels of grain, assuming $\frac{4}{5}$ bushel to a cubic foot?

19. A man sold articles for $\frac{17}{20}$ of the cost, receiving for them \$255. What was the cost?

20. Five ninths is $\frac{2}{3}$ of what fraction?

MULTIPLICATION AND DIVISION.

23. Oral Problems.

Unitary Analysis.

1. If 2 yards of calico cost 16 cents, what will 3 yards cost?

First find the cost of 1 yard.

2. Find the cost of $2\frac{1}{2}$ yards of dress goods at the rate of 40 cents for 4 yards.

3. If 5 men require 40 days to do a piece of work, how long would it take 8 men to do it?

4. A can mow $\frac{3}{4}$ of a field in 9 days. How many days would he require to mow $\frac{2}{3}$ of it?

5. If it requires 160 rods of wire for a fence 4 strands high, how many rods would be needed for a 5-strand fence of the same length?

6. If a man can walk 20 miles in 5 hours, in how many hours can he walk 12 miles?

7. To paint a house requires 4 men 12 days. How long would it take 6 men?

8. A train goes 16 miles in 30 minutes. How many miles will it go in $1\frac{1}{2}$ hours?

9. To build a bridge required the labor of 10 men for 24 days. How many men could complete it in 16 days?

10. If 12 acres produce 36 tons of hay, how many tons will 32 acres produce at the same rate?

Ratio Method.

11. At the rate of 75 cents per dozen bunches, what will be the cost of 4 bunches of rhubarb ?

4 is $\frac{1}{3}$ of a dozen.

12. If a certain amount of hay will last 14 horses $4\frac{1}{2}$ months, how many horses will eat it in $1\frac{1}{2}$ months ?

$4\frac{1}{2}$ months is 3 times $1\frac{1}{2}$ months.

13. What will be the cost of 13 pounds of coffee at the rate of \$ 27.90 for a bag of 130 pounds ?

14. If a 60-foot rail weighs 560 pounds, what will be the weight of a piece 15 feet long ?

15. If 31 acres produce 400 bushels of wheat, what will be the yield of 93 acres at the same rate ?

16. Seven cords of beech produce as much heat as 9 cords of pine. How many cords of pine will produce as much heat as 35 cords of beech ?

17. If 24 reams of paper are used in printing 900 copies of a book, how many reams will be required for 300 copies ?

18. A farmer finds that he has obtained 82 bushels of corn from 30 shocks. What will be the yield from 90 shocks at the same rate ?

19. If 9 cords of wood are required to make 8 tons of paper, how many cords will be required to make 72 tons ?

20. Mr. Freeman paid \$10,000 for a farm of 160 acres. He agrees to sell a portion of it at the rate paid for the whole. What should he receive for 32 acres ?

24. Written Problems.

1. If $28\frac{3}{4}$ acres yield a profit of \$230, what will be the profit on $37\frac{1}{2}$ acres at the same rate ?

$$\begin{array}{rcl}
 28\frac{3}{4} \text{ A. yield} & \$230. & \\
 1 \text{ A. yields} & \frac{\$230}{28\frac{3}{4}}. & \\
 37\frac{1}{2} \text{ A. yield} & \frac{\$230 \times 37\frac{1}{2}}{28\frac{3}{4}}. &
 \end{array}$$

Reducing the mixed numbers to improper fractions, we have,

$$\frac{\$230 \times 4 \times 75}{115 \times 2}. \text{ Cancel.}$$

2. If 16 men require $31\frac{1}{2}$ days to do a piece of work, how long will it take 28 men to do it?

$$\begin{array}{rcl}
 16 \text{ men require} & 31\frac{1}{2} \text{ da.} & \\
 1 \text{ man requires} & 31\frac{1}{2} \text{ da.} \times 16. & \\
 28 \text{ men require} & \frac{31\frac{1}{2} \text{ da.} \times 16}{28}. &
 \end{array}$$

$$\text{Number of days} = \frac{63 \times 16}{2 \times 28}. \text{ Cancel.}$$

3. If $4\frac{1}{4}$ times a certain number is 221, what is $12\frac{3}{4}$ times the same number?

4. Find the cost of 9 pairs of stockings at the rate of \$2.80 per dozen pairs.

5. How many bushels of oats at 32 pounds per bushel will be equal in weight to 2400 bushels of corn weighing 56 pounds per bushel?

6. What is the value of 579 (German) marks in (French) francs, the latter being worth $19\frac{3}{10}$ cents in United States money, and the former $23\frac{1}{2}$ cents?

7. If \$600 yield \$30 interest in a year, how much interest should \$720 yield in the same time?

8. What will be the cost of 7000 pounds of coal at \$4.48 per long ton of 2240 pounds?

9. A train requires 18 hours (running time) to cover a certain distance when going at the rate of $24\frac{1}{2}$ miles per hour. How long will it take if it travels $30\frac{5}{8}$ miles per hour?

10. If $\frac{3}{4}$ of an acre of land shows a profit of \$15.80, what is the profit on $2\frac{1}{4}$ acres at the same rate?

25. Oral Exercises.

1. What is $\frac{1}{2}$ of 12? $\frac{2}{3}$ of 24? $\frac{3}{4}$ of 48? $\frac{1}{5}$ of 100?
2. Six is $\frac{1}{2}$ of what number? ($6 = \frac{1}{2}x$) 8 is $\frac{1}{3}$ of what? 12 is $\frac{1}{4}$ of what? 20 is $\frac{1}{5}$ of what?
3. Sixteen is $\frac{1}{3}$ of what number? 16 is $\frac{2}{3}$ of what? 36 is $\frac{1}{4}$ of what? 36 is $\frac{1}{2}$ of what? 36 is $\frac{2}{4}$ of what? 36 is $\frac{3}{4}$ of what? 60 is $\frac{1}{5}$ of what? 60 is $\frac{2}{5}$ of what? 60 is $\frac{3}{5}$ of what? 60 is $\frac{4}{5}$ of what?
4. Twelve is what part of 24? 8 is what part of 24? 16 is what part of 24? 6 is what part of 24? 18 is what part of 24? 4 is what part of 24? 20 is what part of 24?
5. One fourth is what part of $\frac{1}{2}$? $\frac{1}{3}$ is what part of $\frac{1}{2}$? $\frac{1}{6}$ is what part of $\frac{1}{2}$? $\frac{1}{8}$ is what part of $\frac{1}{2}$? $\frac{1}{10}$ is what part of $\frac{1}{2}$?

26. Oral Problems.

1. A farmer sold $\frac{3}{4}$ of a flock of 72 sheep. How many did he retain?
2. Two thirds of A's farm is under cultivation. The remainder of the farm contains 75 acres. How many acres are under cultivation?
3. A man can do $\frac{2}{3}$ of a piece of work in a day. How many days will it take him to do a piece thrice as great?
4. I insure my house for $\frac{4}{5}$ of its value, or \$1600. What is the value of my house?
5. One ninth of the pupils of a certain class were absent on a stormy day. Twenty-four were present. How many pupils belonged to the class?
6. Mr. Jonas raised 600 bushels of wheat. He sold 450 bushels. What part of his crop did he sell? What part did he retain?

7. In an orchard, three quarters of the trees are apple trees, and the remaining 90 are cherry trees. How many apple trees in the orchard?

8. After traveling $\frac{2}{3}$ of his journey, a passenger has still to go 240 miles. What distance has he already traveled?

9. Seven ninths of the cargo of a vessel consists of 630 tons of wheat. How many tons of cargo in the vessel?

10. If silk sells for $\$ \frac{2}{3}$ per yard, how much can be bought for a dollar?

11. What fraction of an hour is 50 minutes?

12. Change $\frac{5}{8}$ day to hours.

13. A sold a horse at a profit of $\frac{1}{8}$ of its cost, receiving for it \$180. What did it cost?

14. The population of a village has decreased $\frac{1}{10}$ in a year. What was the population a year ago, if the present population is 450?

15. A boy of 10 is $\frac{1}{4}$ as old as his father. The former's age will be what fraction of the latter's in 10 years?

27. Written Problems.

1. A drover sold $\frac{6}{11}$ of a drove of 484 sheep. How many sheep did he retain?

2. Four sevenths of a plantation is under cultivation. The remainder of the plantation contains 375 acres. How many acres are there in the plantation?

3. Three men can do $\frac{1}{6}$, $\frac{1}{7}$, and $\frac{1}{8}$, respectively, of a certain piece of work in a day. What part of the work can the three do together in a day? How long will it take the three, working together, to do the whole work?

4. My house is damaged by fire to the extent of $\frac{4}{11}$ of its value. The damage amounts to \$760. What was the original value of the house?

5. Three seventeenths of the pupils of a school were absent. The number present was 168. How many pupils belonged to the school?

6. Of a crop of 594 bushels of corn 451 bushels were sold. What part of the crop was sold?

7. One third of the trees in an orchard are apple trees, two thirds of the remainder are peach trees, and the rest are cherry trees. There are 76 cherry trees. How many apple trees in the orchard?

8. A man has gone $\frac{2}{5}$ of his journey by boat, and 225 miles by rail. He has $\frac{3}{20}$ of the distance yet to travel. What is the length of his trip?

9. A vessel's cargo consists of 405 tons of wheat, 270 tons of ore, and the remainder of hay, the last being one sixth of the cargo. How many tons of hay are there?

10. If velvet is worth $\$3\frac{3}{8}$ per yard, what part of a yard can be purchased for 75 cents?

REVIEW OF DECIMALS.

Sight Exercises.

28. Reduce to common fractions:

1. .5	5. .25	9. .125	13. .375
2. .6	6. .08	10. .16	14. .155
3. .60	7. .625	11. .36	15. .024
4. .84	8. .002	12. .875	16. .075

29. Reduce to decimals:

1. $\frac{1}{4}$	5. $\frac{1}{8}$	9. $\frac{1}{20}$	13. $\frac{5}{8}$
2. $\frac{1}{50}$	6. $\frac{2}{25}$	10. $\frac{3}{5}$	14. $\frac{6}{25}$
3. $\frac{3}{20}$	7. $\frac{3}{4}$	11. $\frac{7}{50}$	15. $\frac{4}{50}$
4. $\frac{3}{8}$	8. $\frac{11}{20}$	12. $\frac{7}{8}$	16. $\frac{3}{40}$

30. Multiply:

1. $300 \times .05$

2. $484 \times .25$

NOTE. When one of the factors is .25, .125, etc., the equivalent common fraction should generally be employed.

3. $200 \times .06$

9. $.375 \times 80$

15. $444 \times .25$

4. $250 \times .4$

10. $.25 \times 84$

16. $848 \times .5$

5. $300 \times .022$

11. $.125 \times 16$

17. $48 \times .625$

6. $500 \times .12$

12. $.75 \times 36$

18. $24 \times .875$

7. $120 \times .24$

13. 1.5×18

19. $2.4 \times .875$

8. $150 \times .24$

14. $.15 \times 18$

20. $.48 \times .625$

31. Divide:

1. $12 \div .03$

2. $24 \div .6$

$12 \div .03 = 1200 \div 3.$ Why?

$24 \div .6 = 240 \div 6.$

3. $22 \div 25$

4. $15 \div 24$

$22 \div 25 = \frac{22}{25} = \frac{?}{100}.$

$15 \div 24 = \frac{15}{24} = \frac{5}{8} = \frac{?}{1000}.$

5. $12 \div .25$

$12 \div .25 = 12 \div \frac{1}{4} = 12 \times 4.$

6. $33 \div .375$

$33 \div .375 = 33 \div \frac{3}{8} = 33 \times \frac{8}{3} = 11 \times 8.$

7. $8 \div .02$

12. $15 \div 50$

17. $11 \div .25$

8. $18 \div .06$

13. $24 \div 32$

18. $22 \div .25$

9. $24 \div .4$

14. $16 \div 64$

19. $36 \div .75$

10. $88 \div .022$

15. $18 \div 20$

20. $21 \div .125$

11. $54 \div .12$

16. $30 \div 100$

21. $12 \div .375$

Written Exercises.**32. Add:**

NOTE. Change common fractions to decimals.

1. $16\frac{1}{3} + 84.7 + 96\frac{7}{8} + 75\frac{2}{5}$

2. $.3 + \frac{2}{5} + 7\frac{1}{12} + 35.309 + .0483$

Add:

$$3. \ 3\frac{17}{500} + .0087 + 35.348 + .0907 + \frac{1}{80}$$

$$4. \ .945 + 34.8 + 9.48 + \frac{11}{100} + 826$$

33. Subtract:

$$1. \ 15.36 - .89$$

$$7. \ 6.51 - 3.429$$

$$2. \ 18.45 - 9.7$$

$$8. \ 25.2 - 7.625$$

$$3. \ 126.344 - 85.9$$

$$9. \ 55.007 - 4.26$$

$$4. \ .90072 - .086$$

$$10. \ 89.4 - 83.576$$

$$5. \ 17.09 - 3.25$$

$$11. \ 103.05 - 8.9306$$

$$6. \ 809.72 - 48.4$$

$$12. \ .6 - .4725$$

34. Multiply:

$$1. \ 46.78 \text{ by } .93$$

NOTE. In multiplication of decimals, as well as in all other examples, a pupil should estimate the probable answer before performing the operation. The answer to the foregoing should be less than 46, for .93 is less than 1.

The number of decimal places in the product is equal to the number in the multiplicand added to the number in the multiplier.

$$2. \ 9.76 \times 15.4$$

$$9. \ 7.37 \times .648$$

$$3. \ 37.68 \times 2.234$$

$$10. \ 47 \times .8 \times .2$$

$$4. \ .98 \times .98$$

$$11. \ .126 \times 48$$

$$5. \ 30.75 \times .46$$

$$12. \ 33.343 \times 2.95$$

$$6. \ 7.08 \times .096$$

$$13. \ 8.053 \times 1.47$$

$$7. \ .09 \times 4.56$$

$$14. \ 76.2 \times 4.86$$

$$8. \ .0364 \times .82$$

$$15. \ 14.9 \times .83$$

35. 1. Divide 217.32 by .6.

The divisor .6 is changed to 6 by multiplying it by 10. To multiply 217.32 by 10, the decimal point is moved one place to the right, which makes it 2173.2.

$$\begin{array}{r} 6 \overline{)2173.2} \\ 362.2 \end{array} \text{ Ans.}$$

To divide by a decimal, make the divisor a whole number by removing the decimal point, and make a corresponding change in the dividend. The number of decimal places in the quotient will be equal to the number in the dividend as changed.

2. Divide 1301.3 by .077.

77 thousandths is changed to a whole number by multiplying by 1000, which moves the decimal point 3 places to the right. To make the same change in the dividend, two ciphers must be annexed, making the new dividend 1301300.

$$\begin{array}{r} 16900 \text{ Ans.} \\ 77 \overline{)1301300} \\ \underline{77} \\ 531 \\ \underline{462} \\ 693 \\ \underline{693} \end{array}$$

3. Divide 3.576 by 800.

The divisor in this case should be changed to 8 by dividing it by 100. A corresponding change in the dividend will be made by moving the decimal point two places to the left.

$$\begin{array}{r} 8 \overline{).03576} \\ .00447 \end{array} \text{ Ans.}$$

36. Divide:

1. $384 \div 3.2$

5. $.21504 \div 9.6$

2. $2304 \div .48$

6. $21.504 \div 240$

3. $53.95 \div 83$

7. $3.2 \div .064$

4. $53.95 \div .65$

8. $.432 \div .012$

Divide:

- | | |
|-----------------------|------------------------|
| 9. $2.436 \div 5.8$ | 15. $6.5772 \div 8.4$ |
| 10. $68.6 \div .049$ | 16. $511.344 \div .67$ |
| 11. $2.4 \div .096$ | 17. $734.4 \div .6$ |
| 12. $180 \div 1.5$ | 18. $371.448 \div .77$ |
| 13. $142.4 \div .4$ | 19. $18.612 \div .09$ |
| 14. $105.28 \div 5.6$ | 20. $38.76 \div 3.4$ |

37. Oral Problems.

1. At \$.125 each, what will be the cost of 88 books ?
 $\$ \frac{1}{8} \times 88$
2. A rectangular field is 8.8 rods long and 25 rods wide. How many square rods does it contain ?
Number of square rods in area = number of rods in length \times number of rods in width.
3. What decimal part of 32 is 8 ?
4. How many hours are there in .625 day ?
5. How many nails can be made from a piece of wire 125 inches in length, if each nail requires 2.5 inches ?
6. At 2.15 pounds per quart, what is the weight of 10 gallons of milk ?
7. The quotient is 2.31, the divisor is .3. What is the dividend ?
8. If wool costing 20¢ per pound loses .2 of its weight in cleaning, what is the cost per pound of the cleaned wool ?
9. A has .75 as many sheep as B. The former has 24 sheep. How many has B ?
10. What will be the cost of 40,000 bricks at \$6.50 per thousand ?
11. A farmer sold .75 of a flock of 36 sheep. How many sheep did he retain ?

12. Six sevenths of a farm is under cultivation. There are 120 acres not under cultivation. How many acres are there in the farm?

13. If a man can do .125 of a piece of work in a day, how many days will it take him to do the whole work?

14. My house is insured for \$1600, which is .8 of its value. What is the value of the house?

15. Of the pupils of a certain class .125 are absent. There are 21 present. How many pupils belong to the class?

16. Seventy-five hundredths of the trees in an orchard are apple trees, and the others are cherry trees. If there are 90 of the latter, how many apple trees are there in the orchard?

17. After selling .16 of his oats, Mr. Davis had 168 bushels remaining. How many bushels had he at first?

18. What decimal part of a day is 15 hours?

19. Change .4 ton to pounds.

38. Written Problems.

1. If goods costing \$268 are sold at a gain of .25 of the cost, what is the selling price?

2. The population of a certain village has increased .25 in a year. It numbers now 560 inhabitants. What was the population one year ago?

3. A person performs a certain journey in 14.75 hours, traveling 3.75 miles per hour. How long would it take him if he traveled at the rate of 5.25 miles per hour?

4. Find the cost of 1575 pounds of hay at \$11 per ton of 2000 pounds.

5. What is the price per ton of hay, if 3750 pounds are sold for \$33 $\frac{3}{4}$?

6. Divide sixty and twelve hundredths times seventy-two hundredths by the sum of thirty-two hundredths and fourteen thousandths.

7. How many steps of 2.5 feet will it take to measure .25 mile? (1 mi. = 5280 ft.)

8. The cost of .1875 of an article is equal to what decimal of the cost of .5 of it?

9. By selling a house for \$2800, the owner lost .3 of the price he paid for it. What did it cost him?

10. Find the value of £146 in United States money, £1 being worth \$4.8665.

11. Change 187.50 German marks to United States money, the mark being worth 23.8 cents.

12. At 19.3 cents for a French franc, how many francs can be bought for \$100?

13. Change 50 kilos to pounds, the kilo being 2.2046 pounds.

14. How many meters, each measuring 39.37 inches, are there in a rod ($16\frac{1}{2}$ ft.)?

15. Find the difference in inches between a kilometer (1000 meters) and $\frac{5}{8}$ mile. (1 mi. = 63,360 in.)

CHAPTER II.

PERCENTAGE.

39. Preliminary Exercises.

Read the following fractions: $\frac{4}{100}$, $\frac{5}{100}$, $\frac{6}{100}$, $\frac{7}{100}$.

Instead of writing these fractions with the denominator 100, we sometimes employ the sign %, which is called **per cent**.

40. The expression 7 *per cent*, therefore, means 7 *hundredths*. It is written 7 %.

We can also write 4 %, 5 %, $2\frac{1}{4}$ %, $5\frac{1}{2}$ %, as decimals, thus: .04, .05, .0225, .055.

41. Read the following per cents as decimals:

- | | | | |
|---------------------|-----------------------|-----------------------|-----------------------|
| 1. 2 % | 6. $12\frac{1}{2}$ % | 11. $6\frac{1}{4}$ % | 16. $7\frac{1}{2}$ % |
| 2. $6\frac{1}{2}$ % | 7. $\frac{1}{4}$ % | 12. $3\frac{1}{2}$ % | 17. $\frac{3}{8}$ % |
| 3. $8\frac{1}{4}$ % | 8. 175 % | 13. $62\frac{1}{2}$ % | 18. $15\frac{1}{4}$ % |
| 4. 10 % | 9. $87\frac{1}{2}$ % | 14. 75 % | 19. $\frac{8}{25}$ % |
| 5. $2\frac{1}{2}$ % | 10. $15\frac{1}{4}$ % | 15. $37\frac{1}{2}$ % | 20. $\frac{1}{2}$ % |

42. Change the following per cents to common fractions in their lowest terms:

- | | | | |
|---------|----------------------|-----------------------|-----------------------|
| 1. 10 % | 6. $12\frac{1}{2}$ % | 11. $37\frac{1}{2}$ % | 16. $6\frac{1}{4}$ % |
| 2. 15 % | 7. $11\frac{1}{3}$ % | 12. 50 % | 17. $62\frac{1}{2}$ % |
| 3. 25 % | 8. 35 % | 13. 150 % | 18. $\frac{1}{2}$ % |
| 4. 75 % | 9. 45 % | 14. $66\frac{2}{3}$ % | 19. $\frac{3}{4}$ % |
| 5. 90 % | 10. 125 % | 15. $1\frac{1}{3}$ % | 20. $\frac{5}{8}$ % |

43. 1. Express $\frac{1}{2}$ as a per cent.

$$\frac{1}{2} = \frac{50}{100} = 50\%$$

2. Express $\frac{3}{8}$ as a per cent.

$$\frac{3}{8} = \frac{37\frac{1}{2}}{100} = 37\frac{1}{2}\%$$

To change a common fraction to an equivalent per cent, reduce the given fraction to a fraction whose denominator is 100, and replace the denominator by the sign %.

Oral Exercises.

44. Express the following fractions as per cents:

- | | | | | |
|------------------|------------------|-------------------|--------------------|--------------------|
| 1. $\frac{1}{4}$ | 5. $\frac{5}{8}$ | 9. $\frac{3}{5}$ | 13. $\frac{1}{5}$ | 17. $\frac{7}{10}$ |
| 2. $\frac{3}{4}$ | 6. $\frac{7}{8}$ | 10. $\frac{4}{5}$ | 14. $\frac{5}{6}$ | 18. $\frac{9}{10}$ |
| 3. $\frac{1}{8}$ | 7. $\frac{1}{5}$ | 11. $\frac{1}{3}$ | 15. $\frac{1}{10}$ | 19. $\frac{3}{20}$ |
| 4. $\frac{3}{8}$ | 8. $\frac{2}{5}$ | 12. $\frac{2}{3}$ | 16. $\frac{3}{10}$ | 20. $\frac{1}{40}$ |

45. Read the following decimals as per cents:

1. .125

As a decimal, this is read 125 thousandths; as a per cent, it becomes $12\frac{1}{2}$ per cent, or 12 and 5 tenths per cent.

2. .005

This is read $\frac{1}{2}$ per cent, or $\frac{1}{2}$ of 1 per cent.

3. 4.75

This is read 475 per cent.

- | | | |
|---------|------------|-----------|
| 4. .15 | 8. .075 | 12. 3.125 |
| 5. .9 | 9. .0625 | 13. .8 |
| 6. .375 | 10. .0075 | 14. .42 |
| 7. 2.4 | 11. .00625 | 15. 4.2 |

46. The portion of arithmetic that deals with per cents is called **percentage**.

The applications of percentage include no principles that have not already been studied in the applications of fractions and of decimals.

47. The three leading types of problems in percentage are shown in the following examples :

1. Find 20% of 150.
2. Thirty is 20% of what number ?
3. Thirty is what per cent of 150 ?

48. As examples in fractions, these have already been worked in the following form :

1. Find $\frac{1}{3}$ of 150.
2. Thirty is $\frac{1}{3}$ of what number ?
3. Thirty is what fraction of 150 ?

49. The corresponding decimal examples would be the following :

1. Find .2 of 150.
2. Thirty is .2 of what number ?
3. Thirty is what decimal of 150 ?

50. In each of the examples in percentage (Art. 47), there are three terms: the *base*, the *rate*, and the *percentage*. Two of these are given, from which the third is to be calculated. In 1, the percentage is required; in 2, the base; in 3, the rate.

TO FIND THE PERCENTAGE.

51. Sight Exercises.

Find :

- | | |
|----------------------------------|------------------------------------|
| 1. 50 % of 16 bushels | 6. $16\frac{2}{3}$ % of 102 miles |
| 2. $12\frac{1}{2}$ % of 24 books | 7. $33\frac{1}{3}$ % of 84 yards |
| 3. 20 % of \$300 | 8. 10 % of 25 feet |
| 4. 80 % of 45 sheep | 9. $66\frac{2}{3}$ % of 33 gallons |
| 5. $37\frac{1}{2}$ % of \$80 | 10. 75 % of \$120 |

Find:

- | | |
|--------------------------------------|----------------------------------|
| 11. 25 % of 36 inches | 16. $116\frac{2}{3}$ % of \$600 |
| 12. $\frac{1}{2}$ % of \$72 | 17. 125 % of 44 feet |
| 13. 150 % of 20 inches | 18. 40 % of \$1000 |
| 14. $\frac{1}{5}$ % of \$240 | 19. $87\frac{1}{2}$ % of 64 rods |
| 15. $133\frac{1}{3}$ % of 150 pounds | 20. $3\frac{1}{3}$ % of \$30 |

52. Written Exercises.

Find the percentage:

1. 52% of \$67.50
- | |
|-----------|
| \$67.50 |
| .52 |
| <hr/> |
| 13500 |
| 33750 |
| <hr/> |
| \$35.1000 |
- In this example, \$67.50 is the *base*, 52 is the *rate* per cent.

Ans. \$35.10

To find the percentage, multiply the base by the rate considered as hundredths.

- 2.
- $37\frac{1}{2}$
- % of 728 acres

$$37\frac{1}{2}\% \text{ of } 728 \text{ acres} = 728 \text{ acres} \times \frac{37\frac{1}{2}}{100} = 728 \text{ acres} \times \frac{3}{8} \quad \text{Cancel.}$$

NOTE. In any example involving multiplication by a decimal, the decimal should be changed to a common fraction when the work will be shortened by such a change.

- | | |
|---|-------------------------------------|
| 3. 14 % of 375 miles | 12. $12\frac{1}{2}$ % of 32.96 rods |
| 4. $\frac{1}{2}$ % of \$715 | 13. 50 % of 1984 cords |
| 5. 20 % of 1736 miles | 14. $\frac{1}{2}$ % of 1984 gallons |
| 6. 25 % of 1384 acres | 15. $2\frac{1}{2}$ % of 1260 yards |
| 7. $16\frac{2}{3}$ % of \$44.64 | 16. $37\frac{1}{2}$ % of 464 books |
| 8. $13\frac{1}{3}$ % of 802 $\frac{1}{2}$ bushels | 17. 3 % of 1900 pupils |
| 9. 130 % of 88 tons | 18. $1\frac{1}{4}$ % of 6480 pounds |
| 10. $33\frac{1}{3}$ % of 297 cows | 19. $\frac{3}{4}$ % of \$76 |
| 11. $\frac{4}{5}$ % of \$12,000 | 20. 75 % of \$76 |

53. Oral Problems.

1. A farmer loses 3% of 200 tons of hay by rain. How many tons does he lose?

2. There are 36 pupils belonging to a class. On a stormy day 25% are absent. How many are present?

3. If coffee loses 15% of its weight in roasting, what is the loss in weight of a bag of coffee weighing 130 pounds?

4. A man's yearly income is 4% of \$30,000. What is his income?

5. Last year the population of a city was 25,000. What is the present population if the increase is 4%?

6. A farmer put in his barn 200 tons of hay. What does it weigh after losing 2% by drying?

7. A certain ore produces 8% of metal. How many pounds of metal in 2000 pounds of ore?

8. A merchant loses 3% on the cost of certain goods. What does he lose if the goods cost \$600?

9. In a school of 120 pupils 45% are girls. How many girls in the school?

10. A man has a farm of 200 acres, of which he has 84% under cultivation. How many acres are under cultivation?

11. If $\frac{1}{2}\%$ of commercial lead consists of impurities, how many pounds of pure metal will there be in 2000 pounds of the commercial article?

12. Of a flock of 350 sheep 4% died. How many remained?

13. A girl spelled correctly 98% of 50 words. How many did she miss?

14. Forty-five per cent of an orchard of 200 trees are peach trees. How many peach trees are there?

15. Mr. Jones sold 75% of his crop of 800 bushels of corn. How many bushels did he keep?

16. A man can plow $12\frac{1}{2}\%$ of a field in a day. How many days will it take him to plow the field?

17. If sea water contains $2\frac{3}{5}\%$ of salt, how many pounds of salt can be obtained from 2000 pounds of sea water?

18. A farmer sold all but 25% of his farm of 480 acres. How many acres did he sell?

19. A man who owned $\frac{3}{4}$ of a vessel sold $33\frac{1}{3}\%$ of his share. What part of the vessel did he then own?

20. An agent charged me 2% for selling my house. If the house sold for \$4000, how much should I pay the agent?

21. If a broker receives $\frac{1}{8}\%$ for buying bonds, how much does he receive on a purchase of \$8000?

54. Written Problems.

1. A man receives 4% yearly on a capital of \$37,500. What is his monthly income therefrom?

2. The population of a certain city is 25,425, of whom 8% are of foreign birth. How many are of foreign birth?

3. Hay, weighing 275 tons when placed in the barn, has lost 6% in weight. What is its present weight?

4. If $3\frac{3}{4}\%$ of metal can be obtained from a certain ore, how many pounds of metal are contained in 2240 pounds of this ore?

5. A merchant lost $2\frac{1}{3}\%$ on goods that cost him \$640. What did he receive for them?

6. In a school of 400 pupils, 49% are boys. How many girls in the school?

7. The owner of a farm of 640 acres has $37\frac{1}{2}\%$ under cultivation, 50% in woods, and the remainder in pasture. How many acres are in pasture?

8. If iron loses $\frac{3}{4}\%$ in handling, what will be the loss on 100 tons of 2240 pounds?

9. Of a flock of 400 sheep 4% died, and 25% of the remainder were sold. How many sheep were left?

10. A school of 450 pupils had 98% in attendance. How many were present?

11. Thirty-seven and one half per cent of the trees in an orchard of 464 trees are peach trees, 25% are cherry trees, and the remainder are apple trees. How many apple trees are there?

12. A farmer had 800 bushels of wheat, of which 4% was lost in handling, cleaning, etc. He sold 25% of the remainder at 90¢ per bushel. How much did he receive for the quantity sold?

13. A can do 5% of a piece of work in a day, and B can do $7\frac{1}{5}\%$ of it in a day. How many days will it take both to do the whole work? How many days will it take A to do it alone? B to do it alone?

14. If salt water contains $2\frac{3}{4}\%$ of salt, how many pounds of salt water will be required to produce 2000 pounds of salt?

15. A farmer sold to one purchaser 20% of his farm of 480 acres, and to another 25% of the remainder. How many acres had he remaining?

16. The owner of $\frac{5}{8}$ of a vessel sold 20% of his share for \$ 2250. What is the value of the whole vessel at that rate?

17. An agent sold 600 bushels of wheat at 90¢ per bushel. His charge was 2% on the amount received for the wheat. How much did he return to the owner?

18. A broker receives $\frac{1}{8}\%$ of \$ 17,400 for selling bonds. How much does he receive?

19. A farmer was offered 50¢ per hundredweight for 100 tons of hay. He sold it for 60¢ per hundredweight some months later, after it had lost $6\frac{1}{2}\%$ in weight. Did he gain or lose by refusing the first offer?

20. How many pounds of bread can be made from 200 pounds of flour, if the dough weighs 60% more than the flour, and if the bread weighs 15% less than the dough?

TO FIND THE BASE.

55. Preliminary Exercises.

1. Twelve is $\frac{1}{4}$ of what number? 12 is .25 of what number? 12 is 25% of what number?

2. Thirty-six is $\frac{3}{4}$ of what number? 36 is .75 of what number? 36 is 75% of what number?

3. Forty is $1\frac{1}{4}$ times what number? 40 is $\frac{5}{4}$ of what number? 40 is 1.25 times what number? 40 is 125% of what number?

4. The percentage 40 is 1.25 times what base?

5. The percentage 60 is 2 times what base? The percentage 60 is 200% of what base?

6. When 60 is 2 times a base, how is the base found? What sign is written between 60 and 2 to indicate the operation required to obtain the result?

7. When 12 is $\frac{1}{4}$ of a base, what sign is written between 12 and $\frac{1}{4}$ to indicate the required operation?

8. When 12 is .25 of the base, what sign do we write between 12 and .25 to indicate the required operation?

9. In expressing the operation of finding 25% of 48, how is 25% written?

10. How do we express the operation of finding the base, when 25% of it is 12?

56. Since the percentage is found by *multiplying* the base by the rate expressed as hundredths,

The base is found by dividing the percentage by the rate expressed as hundredths.

57. Indicating the number (or quantity) representing the base by B , the number representing the rate per cent by R , and the number (or quantity) representing the percentage by P , we obtain the following:

$$58. \quad B \times \frac{R}{100} = P. \quad B = P \div \frac{R}{100}.$$

59. Oral Exercises.

1. 21 is 50% of what number?
2. 47 is 10% of what number?
3. 18 is 25% of what number?
4. 15 is $33\frac{1}{3}\%$ of what number?
5. 27 is 9% of what number?
6. 36 is 40% of what number?
7. 42 is $116\frac{2}{3}\%$ of what number?
8. 55 is 125% of what number?
9. 40 is $66\frac{2}{3}\%$ of what number?
10. 72 is $112\frac{1}{2}\%$ of what number?
11. Find $12\frac{1}{2}\%$ of 80 bushels.
12. 80 books is $12\frac{1}{2}\%$ of how many books?
13. What is 20% of 400 sheep?
14. \$400 is 20% of what sum?
15. 4 miles is 40% of how many miles?
16. What is $\frac{1}{3}\%$ of \$40?
17. \$40 is $\frac{1}{3}\%$ of what?
18. 30 gallons is $37\frac{1}{2}\%$ of how many gallons?
19. 90 rods is 75% of how many rods?
20. 36 inches is 300% of how many inches?

60. Written Exercises.

1. \$35.10 is 52% of what sum?

$$\text{Base} \times .52 = \$35.10 \text{ (Art. 58).}$$

$$\text{Base} = \$35.10 \div \frac{52}{100} = \$35.10 \div .52.$$

Following the rule for division of decimals, the divisor is changed to a whole number, and a corresponding change is made in the dividend.

The quotient, \$67.50, is the base required.

$$\begin{array}{r} \text{\$67.50} \quad \text{Ans.} \\ 52 \overline{) \$3510.} \\ \underline{312} \\ 390 \\ \underline{364} \\ 260 \\ \underline{260} \end{array}$$

To find the base, divide the percentage by the rate considered as hundredths.

2. 273 acres is
- $37\frac{1}{2}\%$
- of how many acres?

$$\text{Base} \times .37\frac{1}{2} = 273 \text{ acres.}$$

$$\text{Base} = 273 \text{ acres} \div .37\frac{1}{2} = 273 \text{ acres} \div \frac{3}{8}.$$

To divide 273 acres by $\frac{3}{8}$, the divisor is inverted, according to the rule for division of fractions.

$$\text{Base} = 273 \text{ acres} \times \frac{8}{3}. \quad \text{Cancel.}$$

3. 214 is 25% of what number?

$$\text{Base} = 214 \div \frac{1}{4}.$$

Although the pupil obtains the result by multiplying the percentage by 4, he has really divided 214 by $\frac{1}{4}$. Using the decimal .25, the operation would be indicated

$$\text{Base} = 214 \div .25,$$

while the pupil would be expected to obtain the result through the short method of multiplying 214 by 4.

4. Percentage 385 miles; rate 14%. Base?
5. Rate $\frac{1}{2}\%$; percentage \$3.75. Base?
6. Base 1728 cu. in.; rate $37\frac{1}{2}\%$. Percentage?
7. Rate $6\frac{1}{4}\%$; base \$89.28. Percentage?
8. Rate $13\frac{1}{3}\%$; percentage 105 bushels. Base?
9. Percentage 88 tons; rate $16\frac{2}{3}\%$. Base?

10. Base 165 cows: rate $33\frac{1}{3}\%$. Percentage?
11. Rate $37\frac{1}{2}\%$: base 928 volumes. Percentage?
12. Percentage 24.125 sq. rd.: rate $12\frac{1}{2}\%$. Base?
13. Base 8.75 gallons: rate 160%. Percentage?
14. Rate $1\frac{1}{3}\%$: percentage 475 bushels. Base?
15. Percentage \$475: rate $\frac{2}{3}\%$. Base?

61. Oral Problems.

1. How many tons of hay has a farmer, if his loss of 3% by rain is 12 tons?
2. On a stormy day 24 pupils are present, or 80% of the class. How many pupils belong to the class?
3. A man gains 20% a year on the sum invested in his farm. What is the investment if his gain is \$1200?
4. The population of a certain city increased 1500 in a year, which was a gain of 3%. What was its population the previous year?
5. How many pounds of ore, yielding 6% of zinc, will produce 120 pounds of zinc?
6. How much did I borrow, if 5% of the sum borrowed is \$20?
7. Of the pupils of a certain school the boys constitute 48%. How many girls in the school, if there are 48 boys?
8. A dealer's loss of 3% on the sum paid for cattle amounts to \$60. How much did he pay for the cattle?
9. How much did a seller receive for goods, if his loss on them at 4% amounted to \$40?
10. A farmer lost 20% of his wheat by fire. What per cent of it had he left?
11. A farmer, losing 20% of his wheat, had 80 bushels remaining. How many bushels had he at first?

12. Mr. N lost by fire 40 bushels of a crop of 200 bushels of wheat. What per cent of it did he lose?

13. Of a crop of 300 bushels of wheat a farmer saved 270 bushels. What per cent did he lose?

14. A man received \$330 for a team of horses that cost him \$300. What per cent did he make on the cost?

62. Written Problems.

1. A dealer loses $2\frac{1}{3}\%$ of a lot of coal in handling, screening, etc. How many tons has he bought, if the loss is 203 tons?

2. Mr. Jones's cotton crop was $77\frac{1}{2}\%$ of last year's. Last year's yield was 60 bales of 400 pounds each. How many pounds did he raise this year?

3. How many bushels of wheat did a farmer raise this year, if last year's crop of 504 bushels was 84% of this year's crop?

4. A man receives annually, as rent, $10\frac{1}{2}\%$ of the cost of a house. What did the house cost him, if his tenant pays him \$17.50 per month?

5. After traveling 385 miles, a passenger has completed 14% of his journey. How many miles has he yet to travel?

6. The Evergreen Dairy sold 15% more milk this year than last. The sales last year were 10,200 quarts. How many quarts were sold this year?

7. M can do $\frac{1}{30}$ of a piece of work in a day. What part of it can N do in a day, if he can do 20% more work than M?

8. John requires 30 days to do a piece of work that James can do in 25 days. John does what per cent less work in a day than James?

9. In washing a certain quantity of wool there was a loss of weight of 17%. What did the unwashed wool weigh, if the weight after washing was 332 pounds?

10. A farmer sold 20% of his wheat to A, and 20% of the remainder to B, and had 360 bushels remaining. How many bushels did he sell to B?

TO FIND THE RATE.

63. Preliminary Exercises.

1. Twelve is what fraction of 48? 12 is what decimal of 48? 12 is what per cent of 48?

2. Thirty-six is what fraction of 48? 36 is what decimal of 48? 36 is what per cent of 48?

3. Forty is how many times 32? 40 is what fraction of 32? 40 is how many hundredths of 32? 40 is what per cent of 32?

4. The percentage 40 is how many times the base 32?

5. The percentage 60 is how many times the base 30?
60 is what per cent of 30?

64. Oral Exercises.

1. What per cent of 50 is 40?

2. 50 is what per cent of 40?

3. 15 is what per cent of 3?

4. What per cent of 15 is 3?

5. What per cent of $4\frac{1}{2}$ is $1\frac{1}{2}$?

6. $\frac{2}{3}$ is what per cent of $\frac{5}{3}$?

7. $\frac{1}{3}$ is what per cent of $\frac{1}{6}$?

8. What per cent of 75 is 25?

9. What per cent of 10 is 11?

10. 120 is 90 increased by what per cent of 90?

11. 90 is 120 decreased by what per cent of 120?

12. 3 is what per cent of 4?
13. $3 + 1$ is what per cent of $4 + 1$?
14. $3 - 1$ is what per cent of $4 - 1$?
15. Twice 3 is what per cent of twice 4?

65. Written Exercises.

1. \$35.10 is what per cent of \$67.50?

$$\$67.50 \times \frac{R}{100} = \$35.10. \quad (\text{Art. 58.})$$

$$\frac{R}{100} = \$35.10 \div \$67.50.$$

Following the rule for division of decimals, the divisor is changed to a whole number, and a corresponding change is made in the dividend.

The result .52 gives the rate expressed as hundredths. The answer, therefore, is 52 %.

$$\begin{array}{r} .52 \\ \$675 \overline{) \$351.0} \\ \underline{337 \ 5} \\ 13 \ 50 \\ \underline{13 \ 50} \end{array}$$

To find the rate, divide the percentage by the base, expressing the result as hundredths.

To obtain the answer directly in terms of the per cent, we may proceed as follows:

$$\begin{aligned} \frac{R}{100} &= \frac{\$35.10}{\$67.50} \\ R &= \frac{\$35.10 \times 100}{\$67.50} \end{aligned}$$

Strike out the decimals in the numerator and the denominator. Cancel. The result gives the *number* that indicates the rate per cent, viz., 52.

2. 273 acres is what per cent of 728 acres?

Writing this in the form of an equation, we have:

$$273 A = \frac{?}{100} \text{ of } 728 A.,$$

$$\text{or} \quad \frac{R}{100} \times 728 = 273,$$

from which we obtain

$$R = \frac{273 \times 100}{728}. \quad \text{Cancel.}$$

Difficulty in determining which number is the base may be avoided by writing the problem in the form of an equation, as given above.

3. 225 is what per cent of 15 ?

$$225 = \frac{R}{100} \times 15,$$

or

$$\frac{R}{100} \times 15 = 225,$$

$$\frac{R}{100} = \frac{225}{15}.$$

$$R = \frac{22500}{15}. \quad \text{Ans. } 1500 \%.$$

NOTE. Some pupils think a result of 1500 per cent looks too large, and they frequently make the answer 15 per cent unless they are carefully drilled.

4. What per cent of \$750 is \$3.75?

$$\frac{R}{100} \times 750 = 3.75.$$

$$\frac{R}{100} = \frac{3.75}{750}.$$

$$R = \frac{3.75 \times 100}{750} = \frac{375}{750} = \frac{1}{2}. \quad \text{Ans. } \frac{1}{2} \text{ per cent.}$$

5. 385 miles is what per cent of 2750 miles?
6. What per cent of 385 miles is 2750 miles?
7. 144 cu. in. is what per cent of 1728 cu. in.?
8. One fourth of a day is what per cent of 18 hours?
9. What per cent of a ton is 1425 pounds?
10. 108 bushels is what per cent of 160 bushels?
11. \$8.75 is what per cent of \$70?
12. \$8.75 is what per cent of \$700?
13. \$8.75 is what per cent of \$7000?
14. \$8.75 is what per cent of \$7?
15. 4 gallons is what per cent of 225 gallons?
16. What per cent of 165 cows are 66 cows?
17. What per cent of $\frac{3}{4}$ is $\frac{3}{4}$?
18. $\frac{2}{3}$ is what per cent of $\frac{3}{4}$?
19. $1\frac{3}{7}$ is what per cent of $2\frac{1}{2}$?
20. $2\frac{1}{2}$ is what per cent of $1\frac{3}{7}$?

66. Oral Problems.

1. A girl solved 9 problems out of 10. What per cent did she solve?
2. Out of 200 eggs 190 were good. What per cent of the eggs were bad?
3. What is the percentage of attendance, when 24 pupils are present out of a class of 25?
4. What per cent of a class of 32 are boys, if there are 20 girls in the class?
5. A man who borrowed \$300 pays \$18 per year for the use of it. What per cent does he pay?
6. A dealer bought 200 bushels of corn. In selling it in small quantities, he lost 4 bushels by waste, etc. What per cent did he lose?
7. A farm 200 rods long is 160 rods wide. The width is what per cent of the length?
8. The owner of $\frac{5}{6}$ of a boat sold $\frac{1}{6}$. What per cent of his share did he sell?
9. R owned $\frac{3}{4}$ of a field; he sold $\frac{1}{3}$ of his share. What per cent of the field did he sell?
10. A boy sells at \$1.25 a bushel apples that cost him \$1 a bushel. What per cent of his receipts does he gain?
11. The population of a village increases from 800 to 900. What is the per cent of increase?
12. If the population decreases from 900 to 800, what is the per cent of decrease?

67. Written Problems.

1. A sells $\frac{1}{10}$ of his land, then $\frac{1}{9}$ of the remainder, then $\frac{1}{8}$ of the remainder. What per cent of the farm is unsold?
2. An agent receives \$350 for selling 350 acres of land at \$50 per acre. What per cent of the selling price does he receive?

3. Mr. Jones collects a bill of \$436.50, for which he charges his employer \$8.73. What per cent does he charge for making the collection?

4. The weight of an ox increased from 1200 pounds to 1476 pounds. What per cent did it increase?

5. A quantity of wool weighing 380 pounds weighed 304 pounds after cleaning. What per cent did it lose?

6. A baseball club wins 46 games and loses 34 games. What per cent of the games does it win?

7. In making a trip of 800 miles, a man travels 620 miles by boat and the remainder by rail. What per cent of the trip is made by rail?

8. The weight of the letters on a certain route is 175 tons and of the other mail matter 1125 tons. What per cent of the total weight is the weight of each?

9. To insure a house for \$3675, the owner pays \$12.25. What per cent of the amount insured does he pay?

10. From 240 tons of sugar cane there were extracted 174 tons of juice. What per cent of the whole weight was the weight of the juice?

11. Of a farm of 320 acres, 24 acres are planted in fruit. What per cent of the farm is in fruit?

12. By draining a certain field, a farmer increases the yield of hay from $37\frac{1}{2}$ tons to 50 tons. What per cent is the increase?

13. Two different fertilizers are applied to the same quantity of land. On one the yield is 320 bushels of oats, weighing 30 pounds to the bushel; on the other the yield is 400 bushels, weighing 32 pounds to the bushel. By what per cent does the weight of the latter crop exceed that of the former?

14. A piece of beef weighing $18\frac{3}{4}$ pounds before cooking weighs 15 pounds when roasted. What per cent of its weight is lost?

15. How many pounds of bread can be made from a bushel of rye weighing 56 pounds, if the flour weighs 25% less than the grain and the bread weighs $33\frac{1}{3}\%$ more than the flour?

AMOUNT AND DIFFERENCE.

68. Preliminary Exercises.

1. Multiply 15 by $1\frac{1}{5}$. Increase 15 by $\frac{1}{5}$ of itself. Find $\frac{6}{5}$ of 15.

2. Diminish 15 by $\frac{1}{5}$ of itself. Find $\frac{4}{5}$ of 15.

3. If $\frac{6}{5}$ of a number is 18, what is the number? What number increased by $\frac{1}{5}$ of itself gives 18 as the result?

4. What number diminished by $\frac{1}{5}$ of itself gives a remainder of 12?

5. Fifteen increased by 20% of itself equals what? 18 equals what number increased by 20% of itself?

69. Amount = Base + Percentage.
Difference = Base — Percentage.

70. Sight Exercises.

1. Increase 18 by 50% of itself. Amount?

2. Diminish 150 by $33\frac{1}{3}\%$ of itself. Difference?

3. Amount 27; rate 50%. Base?

4. Difference 100; rate $33\frac{1}{3}\%$. Base?

5. What number increased by 25% of itself becomes 45?

6. What number decreased by 50% of itself becomes 18?

7. A number increased by $12\frac{1}{2}\%$ of itself becomes 108. What is the number?

8. Difference 45; rate $16\frac{2}{3}\%$. Base?

9. What number increased by 20% of itself becomes 420?

10. Difference 28; rate $12\frac{1}{2}\%$. Base?
11. A number increased by 75% of itself becomes 140. Find the number.
12. What number diminished by 25% of itself becomes 150?
13. Amount 28; rate $16\frac{2}{3}\%$. Base?
14. A number diminished by $33\frac{1}{3}\%$ of itself becomes 60. Find the number.
15. Rate $66\frac{2}{3}\%$; amount 60. Base?
16. What number increased by 25% of itself becomes 75?
17. What number diminished by 75% of itself becomes 120?
18. Rate $87\frac{1}{2}\%$; difference 300. Base?
19. What number increased by 50% of itself becomes 300?
20. Difference 140; rate $12\frac{1}{2}\%$. Base?

71. Written Problems.

1. What number increased by 27% of itself equals 508?

In this example the amount, 508, is given and the rate, 27% , from which the base is to be obtained. The amount, 508, equals the number + 27% of the number, or 127% of the number, which is 1.27 times the number.

If 1.27 times the number = 508,

the number = $508 \div 1.27$.

$$\begin{array}{r} 400 \\ 127 \overline{)50800} \\ \underline{508} \end{array}$$

Make the divisor a whole number, and make a corresponding change in the dividend.

Ans. 400.

To find the base when the amount and the rate are given, divide the amount by 1 added to the rate considered as hundredths.

2. What number diminished by 19% of itself equals 324?

In this example 324 represents the difference. Number = $324 \div .81$.

To find the base when the difference and the rate are given, divide the difference by 1 diminished by the rate considered as hundredths.

3. A certain number increased by $33\frac{1}{3}\%$ of itself equals 576. Find the number.

$$576 \div 1.33\frac{1}{3} = 576 \div \frac{4}{3}.$$

4. After losing 15% of its weight in roasting, a bag of coffee weighs $110\frac{1}{2}$ pounds. What was its weight before roasting?

5. A city has increased in a year 15% in population. The present population is 32,430. What was the population the year before?

6. A drover sold cattle at an average of \$58.80, which was an increase of 40% over the price he received for another lot. What did he receive per head for the latter?

7. In a year the number of manufacturing establishments has increased 4%, the present number being 215,878. How many were there the year before?

8. In a year the employees increased 16% in number, there being now 5,469,429. What was their number the previous year?

9. A farm was sold for \$11,340, an advance of 40% of its cost. How many dollars more than its cost did the seller receive?

10. In a certain school the number of girls is 10% less than the number of boys. There are 279 girls. How many pupils are there in the school?

11. A piece of property was sold for \$8000, which was 150% in excess of its cost. What was the cost?

12. The yield of an orchard was 405 bushels, which was 19% less than the yield of the preceding year. How many bushels did it yield the year before?

13. What number decreased by 26% of itself becomes 262.32 ?

14. A man loses $17\frac{1}{2}\%$ of the cost of a piece of cloth by selling it for \$122.10. What was the cost ?

15. After selling 68% of his sheep, a farmer still had 80. How many had he originally ?

16. I sold a horse for \$180, which was 20% less than its cost. What did I lose by the sale ?

17. I sold a horse for \$180, which was 20% more than its cost. What was my profit ?

18. What was a dealer's net gain or loss on two horses sold at \$150 each, on one of which he gained 25%, and on the other he lost 25% ?

19. After selling 77% of his crop of potatoes, a farmer has 529 bushels left. How many bushels in the crop ?

20. A man traveled $62\frac{1}{2}\%$ of his journey by train, and the remainder, 243 miles, by steamer. How many miles did he travel ?

72. Oral Problems.

1. A city increased 10% in population in a year. It then had 22,000 inhabitants. How many had it the previous year ?

2. A dealer gained 25% by selling a book for \$1.50. What did it cost ?

3. After selling 20% of his chickens, John has 20 left. How many had he at first ?

4. The minuend is 50. The remainder is 25% of the subtrahend. What is the remainder ?

5. What number increased by 75% of itself equals $12\frac{1}{4}$?

6. William has 20 apples; he has 25% more apples than John. How many has John ?

7. John has 16 apples; he has 20% fewer apples than William. How many has William?

8. A lot was sold for \$600, which was 200% more than it cost. What did it cost?

9. Twenty per cent of a farmer's cows are Jerseys; the remaining 24 are Ayrshires. How many cows has he?

10. There are 2 more boys than girls in a class in which 45% of the pupils are girls. How many pupils are there in the class?

73. Miscellaneous Written Drills.

1. Rate 11%; percentage 1001 men. Base?
2. Amount \$287.50; base \$250. Rate?
3. Rate 13%; amount 1469 acres. Percentage?
4. Base 350 bushels; rate 21%. Amount?
5. Rate 16%; base 325 pounds. Difference?
6. Amount \$262.60; rate 102%. Base?
7. Base 416 cows; percentage 728 cows. Rate?
8. Difference 456 tons; rate 24%. Percentage?
9. Percentage 147 sheep; rate $87\frac{1}{2}\%$. Amount?
10. Base 125 bales; rate 48%. Difference?
11. Rate 23%; difference 201 gallons. Base?
12. Percentage 43 miles; amount 243 miles. Rate?
13. Rate 160%; base 225 trees. Percentage?
14. Base \$250; rate 350%. Amount?
15. Percentage 117 horses; rate 39%. Difference?
16. Rate $11\frac{1}{2}\%$; amount \$304.50. Base?
17. Amount 250 rods; base 210 rods. Rate?
18. Difference $297\frac{3}{4}$ oz.; rate $\frac{3}{4}\%$. Percentage?
19. Rate $\frac{7}{8}\%$; percentage $114\frac{2}{7}$. Amount?
20. Base 264 quarts; rate 75%. Difference?

PROFIT AND LOSS.

74. The *profit* or the *loss* in a business transaction is the difference between the cost and the selling price.

When the expense of conducting business or of selling goods is deducted from the profit or added to the loss, the result is the *net* gain or loss.

75. In problems in profit and loss the cost is the *base*, the profit or the loss is the *percentage*, and the part of the cost represented by the profit or the loss expressed in hundredths is the *rate*.

76. When the transaction produces a profit, the selling price is the *amount* ; when it causes a loss, the selling price is the *difference*.

77. Oral Exercises.

1. Cost \$ 24 ; gain 25%. Selling price ?

SUGGESTION. Use a short method wherever practicable.

2. Cost \$1.50; selling price \$ 2. Gain % ?
3. Selling price \$ 240 ; loss 20%. Cost ?
4. Selling price \$ 240 ; gain 20%. Cost ?
5. Cost \$ 30 ; loss \$ 6. Loss % ?
6. Selling price \$ 24 ; loss \$ 6. Loss % ?
7. Gain \$ 50 ; selling price \$ 75. Gain % ?
8. Selling price \$ 75 ; gain $66\frac{2}{3}\%$. Cost ?
9. Selling price \$ 75 ; loss $66\frac{2}{3}\%$. Cost ?
10. Cost 5¢ ; selling price 16¢. Gain % ?

78. Written Exercises.

1. Cost \$ 250 ; gain 28%. Selling price ?
2. Cost \$1.50 ; selling price \$ 1.62 $\frac{1}{2}$. Gain % ?

3. Selling price \$ 182; gain $16\frac{2}{3}\%$. Cost ?
4. Selling price \$ 67.20; loss 16 %. Cost ?
5. Cost \$ 375.96; loss \$ 62.66. Loss % ?
6. Selling price \$ 69; loss \$ 6. Loss % ?
7. Gain \$ 9.76; selling price \$ 133.01. Gain % ?
8. Selling price \$ 93.60; gain $8\frac{1}{3}\%$. Cost ?
9. Selling price \$ 1274.10; loss 7 %. Cost ?
10. Cost \$ 864; selling price \$ 936. Gain % ?

79. Oral Problems.

1. A house that cost \$ 5000 was sold for \$ 6000. What was the gain per cent ?
2. A farm costing \$ 5000 was sold at a gain of 25 %. What was the gain ?
3. If I buy eggs at 30 cents a dozen and sell at a loss of 10 %, at what price do I sell them ?
4. Berries are sold at 24 cents, which is 20 % more than they cost. How much did they cost ?
5. What is the cost of berries sold for 24 cents at a loss of 20 % ?
6. What per cent will be gained by buying beans at 5 cents a quart and selling them at 7 cents ?
7. Cloth costing \$ 1.60 a yard is sold at a gain of $12\frac{1}{2}\%$. What is the selling price ?
8. What per cent is lost, when tea costing 40 cents per pound is sold for 24 cents per pound ?
9. By selling an article for \$ 30, I lost 25 %. What did it cost ?
10. I gained \$ 500 on a house, which was $33\frac{1}{3}\%$ of the cost. How much did it cost ?

80. Written Problems.

1. Wheat bought at 84¢ a bushel is sold at 98¢ . What is the gain per cent?
2. A man bought 75 barrels of apples at $\$2.60$ per barrel and sold them at a gain of 15% . What was the selling price per barrel? What was his total profit?
3. A merchant sold cloth at $\$1.87\frac{1}{2}$ a yard, which was 25% more than it cost. What did it cost?
4. Mr. Rafferty sold a house for $\$2800$, thereby losing 20% . At what price would he have to sell it to gain 20% ?
5. A watch is sold for $\$90$, which is $14\frac{2}{7}\%$ more than it cost. What is the gain?
6. Berries bought at $\$4.80$ a bushel are sold at 18¢ a quart. Find the per cent of gain or of loss.
7. A horse was sold for $\$20$ less than it cost, which was a loss of $13\frac{1}{3}\%$. Find the selling price.
8. By selling an article for $\$6.56$ more than he paid for it, a merchant gained 16% . Find the cost.
9. I sold a horse for $\$175$, which was $12\frac{1}{2}\%$ less than I paid for it. How much would I have gained, if I had sold it for $\$227$?
10. Cloth which cost $\$1.26$ a yard is marked $33\frac{1}{3}\%$ above cost. What is the per cent gain, if it is sold at 10% below the marked price?
11. A speculator bought 5000 bushels of corn at 49¢ a bushel. He sold 40% of it at an advance of 16% , 30% of it at a loss of 10% , and the rest at 50¢ a bushel. Find out how much he gained.
12. A newsdealer bought 400 papers at $\$1.50$ per hundred and sold 376 of them for 2¢ each, the remainder being destroyed. What was his gain per cent?
13. A farm of 430 acres costing $\$20,640$ was sold at a profit of 15% . At what price per acre was it sold?

14. How much did I lose on goods for which I paid \$836, my loss being $6\frac{1}{2}\%$?

15. A merchant sold goods to the amount of \$3650.40, on which his profit was 17%. What was the cost?

16. A farmer sold a piece of land for \$225 more than its cost, gaining $7\frac{1}{2}\%$. What did he receive for the land?

17. A dealer buys 1875 bushels of wheat at 90 cents a bushel. He sells it at \$1 a bushel, after the quantity is diminished 4% by drying out, etc. What per cent does he gain?

18. A grocer buys 400 pounds of tea at 50¢ per pound. He sells $\frac{1}{4}$ of it at a profit of 5¢ per pound, and the remainder at a profit of 10¢ per pound. What is his gain per cent on the whole?

19. A merchant bought 400 yards of cloth at \$1.25 per yard. He sells $\frac{3}{8}$ of it at \$1.30 per yard. At what rate must he sell the remainder to realize a profit of 15% on the entire quantity?

20. Paid $14\frac{1}{2}\%$ per pound for a 130 pound bag of coffee and $\frac{1}{2}\%$ per pound for roasting. The loss in weight is 20%, and the roasted coffee is sold at 21¢ per pound. What is the gain per cent on the total cost of the roasted coffee?

21. A farmer buys 75 sheep, $\frac{1}{3}$ of them at \$4.20 each, $\frac{2}{5}$ of them at \$3.80 each, and the remainder at \$3 each. After spending \$121 for food, care, etc., he sells them at an average of \$6 each. What per cent does he gain on his total outlay?

22. The owner of a farm of 160 acres, which cost him \$40 per acre, sells $\frac{1}{4}$ of it at \$30 per acre and the remainder at \$50 per acre. What per cent does he gain?

23. By the sale of a farm at an advance of \$425 over the cost, the seller makes a profit of $8\frac{1}{2}\%$. How many acres does the farm contain, if the selling price is \$43.40 per acre?

24. A man bought 160 measured bushels of oats, weighing 30 pounds to the bushel, for \$80. He sold them by weight at 56 cents per bushel of 32 pounds. What per cent did he gain or lose?

COMMISSION AND BROKERAGE.

81. When the owner of property desires to sell, he frequently has recourse to a real estate agent. If the agent secures a purchaser, he charges the seller a certain per cent of the selling price as his **commission**.

82. The farmer ships his butter, eggs, berries, etc., to a *commission merchant* to be sold. The latter remits to the farmer the amount obtained, after deducting a commission on this amount, and also any charge that he may have paid for freight or other expense.

83. Large business houses employ persons to buy or to sell goods for them. These *agents*, *factors*, or *salesmen* are usually paid, in whole or in part, by a certain percentage of the value of the goods they buy or sell, as a *commission*, or *brokerage*.

84. A *commission merchant* is usually a person who actually handles the goods consigned to him. He pays the charges for freight, carts the goods to his warehouse or salesroom, and sells them at once at the market rate, unless he is instructed by the consignor to hold them for a better price.

85. The term *broker* is generally applied to an agent who buys or sells any kind of merchandise without actually receiving or handling the commodities. He may buy for a New York shipper 100,000 bushels of wheat stored in a Chicago warehouse or on board of a train on its way to the coast. He merely fixes the price and arranges for the deliv-

ery to the purchaser. His *brokerage* may be a fixed sum per bushel or a percentage of the price, as may be agreed.

86. A person is frequently employed at a commission to collect money due.

87. *Commission*, or *brokerage*, is the sum paid to any person who acts for another as agent, commission merchant, broker, or the like.

88. The expression *net proceeds* indicates the sum remitted to the owner of goods after all expenses attending the sale are deducted, including the commission, or brokerage.

89. In commission, or brokerage, the *base* is the value of goods bought or sold, the amount collected, etc. The *percentage* is the sum paid the agent for his services. The *difference*, or *proceeds*, is the sum received for goods sold less the commission. The *amount* is the cost of goods bought plus the commission for buying.

90. Oral Problems.

1. An agent collected a debt of \$400. What was his commission at 5%?

2. At 2%, what would a commission merchant receive for selling \$120 worth of potatoes?

3. A boy selling eggs at 3% commission made \$1.50. How many eggs at 25 cents a dozen did he sell?

4. How much would a broker receive, at $2\frac{1}{2}\%$, for buying cotton to the value of \$16,000?

5. A man bought through an agent a farm costing \$10,000. What was the sum paid, including the commission at $\frac{1}{2}\%$?

6. What sum was received for goods on which the commission at 4% was \$60?

7. The total cost of an article bought through a broker was \$ 210, including his commission at 5%. What did the broker pay for it?

8. How much would a real estate agent receive for selling a house for \$3000 at a commission of 2%?

9. A collector received \$ 25 for collecting a debt. His commission was $2\frac{1}{2}\%$. What sum did he collect?

10. A commission merchant remits to a farmer \$ 97 after deducting his commission at 3%. For how much were the goods sold?

11. An auctioneer sold goods amounting to \$1000. What were the net proceeds after the commission of 3% and \$20 for other expenses were deducted?

12. How much wheat can be bought through a broker for \$950, when wheat sells for $94\frac{7}{8}$ cents per bushel, and the broker receives $\frac{1}{8}$ cent brokerage per bushel?

13. What would be the commission at $2\frac{1}{2}\%$ on a bale of cotton weighing 480 pounds, sold for 10 cents a pound?

14. A lawyer collected $66\frac{2}{3}\%$ of a debt of \$600. What did he remit to his client after deducting 5% commission?

15. A commission merchant charged a customer \$2100. This included the sum paid for the goods and a commission of 5% on the purchase price. What was the cost of the goods?

91. Written Problems.

1. A broker buys 225 bales of cotton, averaging 480 pounds per bale, at 10.45 cents per pound. What is his brokerage at $2\frac{1}{2}\%$?

$$$.1045 \times 480 \times 225 \times \frac{1}{40}.$$

2. At 4% brokerage, what will a broker receive for selling 250 bags of coffee, weighing 130 pounds each, at 9.55 cents per pound?

3. A commission merchant sold 80 barrels of apples at \$2.75 per barrel. How much did he remit to the owner after deducting his commission of $2\frac{1}{2}\%$?

4. An agent's commission at $2\frac{3}{4}\%$ was \$550 on the sale of 400 acres. At what price did the land sell per acre?

5. A collector charging $2\frac{1}{2}\%$ commission received \$21.50 for collecting 43% of a debt. What sum was collected, and how much does the debtor still owe?

6. A merchant buys through an agent 130 yards of silk at \$1.62 $\frac{1}{2}$ a yard. If he pays $2\frac{1}{4}\%$ brokerage and other expenses amount to \$1.75, what is the lowest price per yard at which the silk can be sold without loss?

7. Mr. Mills bought goods through an agent at a commission of 2%. The cost of the goods added to the commission amounted to \$994.50. What was the commission?

8. A broker buys for me 100 barrels of beef at \$18.25 per barrel. How much per barrel does the beef cost me, including the commission at 3% and other charges amounting to \$15.25?

9. How much do I save by paying a broker $\frac{1}{8}\%$ for buying 400 bushels of wheat at 95 cents a bushel, instead of giving him a commission of $\frac{1}{8}$ cent per bushel?

10. A commission merchant sold for a farmer 560 baskets of peaches. The net proceeds were \$470, after deducting 5% commission and \$8.80 expenses. What did the peaches bring per basket?

11. A broker bought for a produce dealer 560 baskets of peaches at 90 cents a basket. How much per basket did they cost the dealer, if he paid 5% brokerage and \$8.80 for freight, etc.?

12. A broker was paid \$9.75 for buying grain of the value of \$7790.25 at 99 $\frac{7}{8}$ cents per bushel. What was his brokerage per bushel?

13. What are an auctioneer's fees for selling goods to the amount of \$7500 at 5% on the first thousand dollars, $2\frac{1}{2}\%$ on the next four thousand dollars, and 1% on all above five thousand dollars?

14. A collector remitted \$93.60 after deducting $2\frac{1}{2}\%$ of the sum collected. How much did he collect?

15. The net proceeds of a sale of property, after the deduction of the agent's commission of $1\frac{1}{5}\%$ and \$29.52 for advertising and other expenses, were \$8200. What did the property bring at the sale?

INSURANCE.

92. *Insurance* is a contract to pay to a person, or his representatives, a sum of money in the event of a loss.

The two chief classes of insurance are *property insurance* and *personal insurance*.

93. There are various forms of property insurance. *Fire insurance*, the most usual form, is an agreement to indemnify the owner of a building or other property lost by fire. *Marine insurance* covers loss by fire, shipwreck, and acts of piracy. Insurance against losses by theft, cyclone, lightning, and breakage of plate glass windows, may also be obtained.

94. *Life insurance* is the chief form of personal insurance. The essential feature is an agreement to pay some person a certain sum at the death of the person insured. *Endowment insurance* is a combination of investment and simple life insurance, in which the insurer agrees to pay a certain sum at a time named or at the death of the person insured if he dies before that time. *Accident insurance* is an agreement to pay a fixed sum monthly or weekly in case the insured person is injured through accident, and a fixed sum in case death results from an accident.

95. The business of insurance is conducted by incorporated companies. The company issues to the person insured a written contract, termed a *policy of insurance*. This specifies the amount for which the property or life is insured, called the *face of the policy*, the sum to be paid to the company, called the *premium*, and the other details of the agreement.

96. In fire insurance problems the face of the policy is the *base*, the premium is the *percentage*.

NOTE. An insurance policy, which may be obtained from a local agent, should be brought to the classroom and its provisions read and discussed.

97. Oral Problems.

1. What is the cost of insuring furniture for \$750 at 2%?

2. The owner insures his property at $\frac{3}{4}\%$, paying \$15 premium. What is the face of his policy?

3. A merchant insures his goods for \$12,000 at 90 cents per \$100. What is his premium?

4. What rate per cent corresponds to 60 cents on \$100?

5. A house costing \$12,000 is insured for $\frac{3}{4}$ of its value at 1%. What is the premium?

6. I paid \$15 for insuring my house at $\frac{1}{2}\%$. What was the face of my policy?

7. A premium of \$60 at the rate of $1\frac{1}{2}\%$ is paid to insure a store for $\frac{2}{3}$ of its value. What is the value of the store?

8. If a company insures property to the amount of \$800,000 at an average of 1%, and pays losses and expenses amounting to $\frac{3}{4}\%$ of the total amount insured, how much does it gain?

9. My property is insured through an agent for \$10,000 at $\frac{3}{4}\%$. The agent is paid by the company 20% of the premium. How much does the agent receive?

10. Mr. Taylor has insured his property for 20 years for 80% of its value, paying an annual premium of 1%. If the property is worth \$10,000, what are his total payments?

98. Written Problems.

1. Find the premium on a policy for \$8500 at $2\frac{1}{4}\%$.

2. A company insures a house for 3 years upon an advance payment of 2 years' premiums. What advance payment will insure a house worth \$5000 for 80% of its value for 3 years, the rate for one year being $\frac{3}{4}\%$?

3. What will be the total cost of insuring a factory for \$100,000 and the contents for \$35,000 at 60¢ per \$100 on the former, and 90¢ per \$100 on the latter?

4. A shipment of 5000 bushels of corn worth 60 cents per bushel is insured for $\frac{5}{8}$ of its value at $1\frac{1}{2}\%$. What is the premium?

5. What commission does an agent receive for insuring a factory worth \$75,000 for $\frac{2}{3}$ of its value at $1\frac{1}{4}\%$, if the company allows him 20% of the premium?

6. I obtain a policy for 3 years by the payment of double the yearly rate. If the latter is $1\frac{1}{2}\%$, what does my insurance cost me annually on property insured for \$11,250?

7. A building was insured for $\frac{5}{8}$ of its value at 2%, the premium being \$200. What was the value of the building?

8. A storehouse containing 8000 bushels of wheat worth 90 cents a bushel is totally destroyed by fire. If the owner of the wheat is insured for $87\frac{1}{2}\%$ of its value at $\frac{3}{4}\%$, what is his loss, including the premium?

9. A house worth \$5000 is insured for 80% of its value at $1\frac{1}{2}\%$. If the company pays a loss of \$3000 after it has received 24 annual premiums, how much does it pay in excess of the premiums?

10. The owner of a barn pays \$90 premium on the contents at $2\frac{1}{2}\%$, and \$27 on the building at $\frac{3}{4}\%$. What is the total value of the property, if the contents are insured for $\frac{1}{2}$ of their value and the building is insured for $\frac{2}{3}$ of its value?

TAXES AND DUTIES.

99. The nation, the state, the county, the town, the city, the village, the school district — all require money for public uses. The expenses of the national government amount to over a million dollars a day: public improvements, public institutions, and salaries of officials require immense sums of money.

100. Money for public purposes is raised chiefly in two ways:

1. By a tax on property.
2. By a tax on goods manufactured at home and on goods brought into this country.

TAXES.

101. The expenses of states, counties, and all minor subdivisions are met chiefly by a **tax on property**. In some communities a **poll tax** also is levied, which is a tax usually of one or two dollars payable by all male citizens over twenty-one years of age.

102. Property is of two kinds:

1. *Real estate*, or fixed property; such as land, buildings, mines, railroads, etc.
2. *Personal*, or movable, *property*; such as tools, machinery, furniture, jewelry, etc.

103. For purposes of taxation the value of each person's property is determined by men called *assessors*, who are selected for this purpose. The value fixed by the assessors is called the *assessed value* and is generally less than the actual market value.

104. The **tax rate** is then determined by dividing the amount to be raised by taxation by the total assessed value. The rate may be expressed in various ways; it may be given as a per cent, as so many mills or cents on the dollar, as so many dollars on the thousand, etc.

105. Taxes are generally paid to a special officer called a *collector* or *receiver* of taxes, who receives either a fixed salary or a commission on the sum collected.

106. The *base* in tax problems is the assessed value, the tax being the *percentage*.

107. Oral Exercises.

Find the tax :

1. On \$1000 at 4 mills on the dollar.
2. On \$3000 at $\frac{1}{2}\%$.
3. On \$12,000 at \$15 on \$1000.
4. On \$2000 at $4\frac{1}{2}$ mills on the dollar.
5. On \$4000 at 52 cents on \$100.
6. On \$10,000 at 1.4963%.
7. A man's tax bill at $1\frac{1}{2}\%$ is \$30. What is the assessed value of his property?
8. What is the actual value of property assessed at \$2000, which is 40% of its actual value?
9. At how many mills on the dollar must property be taxed to raise \$3000, the assessed value being \$150,000?

10. If my tax is \$ 6.30 at the rate of 7 mills on the dollar, what is the assessed value of my property ?

11. Find the tax on property assessed at \$ 2000, the rate being \$ 11.50 per \$ 1000.

12. The tax on property assessed at \$ 3000 is \$ 13. What is the rate ?

13. Find the collector's fee at 1% on the tax collected on property assessed at \$ 10,000, the tax rate being 1%.

14. What is the assessed value of the property of a person whose tax, at the rate of $5\frac{1}{2}$ mills on the dollar, is \$ 55 ?

15. What rate per cent is equal to the rate of 4 mills on a dollar ?

108. Written Exercises.

1. A tax of $7\frac{1}{2}$ mills on a dollar was levied to build a school costing \$ 3843. What was the assessed value of the property ?

2. What is the tax on \$ 14,600 at $6\frac{1}{5}$ mills on a dollar ?

3. If the rate of taxation is $1\frac{2}{3}\%$, find the tax paid by a company whose property is assessed for \$ 105,000.

4. In a certain town the tax rate is $8\frac{1}{2}$ mills on the assessed value of the property. If property is assessed 75% of its actual value, what will be the tax paid on property worth \$ 25,000 ?

5. Find the tax on a house worth \$ 15,600, if it is assessed at $\frac{2}{5}$ of its actual value and the tax rate is \$ 12.80 on \$ 1000.

6. A town having an assessed valuation of \$ 5,600,000 has to raise by taxation \$ 84,000. What is the tax rate ?

7. A school district wishes to raise by taxation \$ 47,500 for a new schoolhouse. If the assessed value of the property in the district is \$ 2,500,000, what is the tax rate ?

8. My tax last year on an assessment of \$9200 was \$80.50. Find the rate on \$1000.

9. If a piece of property is taxed \$13.75 at the rate of $\frac{1}{2}\%$, what is its assessed value?

10. If the tax rate is 6.7 mills, what is the assessed value of a paper mill on which a tax of \$808.02 is paid?

DUTIES.

109. The income of the national government is derived chiefly from two sources:

1. **Internal revenue**, obtained from a tax on liquors, manufactured tobacco, and certain other articles.

2. Duties on imports.

110. Duties are of two kinds: *specific* and *ad valorem*. The former is a certain rate per pound, yard, gallon, etc. The latter is a per cent of the value of the goods at the place of production. Some articles pay both a specific and an *ad valorem* duty.

111. Duties are paid to *collectors* appointed by the government to receive them. Their offices are called *custom-houses* and are located at *ports of entry*.

112. The importer delivers to the collector a statement of the value of the goods, accompanied by the invoice, or bill, rendered him by the foreign seller. The value is given in the currency of the country from which the goods are received and is verified in this country by *appraisers*, who also indicate the rate of duty to be paid.

Goods subject to specific duties are weighed, measured, or gauged by government officers.

113. The *base* in *ad valorem* duties is the foreign value; the duty is the *percentage*.

114. Oral Exercises.

1. Find the duty on an automobile valued at \$3000, the rate being 45%.
2. At 30¢ per bushel, what will be the duty on 250 bushels of barley imported from Canada?
3. Last year Christmas trees were imported to the value of \$2600. What duty was paid at 20%?
4. Find the duty on 400 bunches of shingles, containing 250 shingles each, at 30¢ per 1000.
5. An importer of toys paid \$105 duty. What was the value of the toys, the rate being 35%?
6. Find the duty at 1.44 cents per pound on 2000 pounds of sugar.
7. On sugar imported from the Philippines the duty is 75% of the regular rate. Find the duty on 1000 pounds, the regular rate being 1.44 cents per pound.
8. Find the duty on 500 pounds of Cuba sugar at 1.44 cents per pound, less a reduction of 20%.
9. What is the duty on 100 square yards of Brussels carpet valued at \$120, the rate being 44 cents a square yard and 40%?
10. Find the duty on 100 square yards of dress goods worth 14 cents a square yard, at 7 cents a square yard and 50%.

115. Written Problems.

1. A dealer imported 40 shotguns at \$4 each and 5 at \$60 each. What was the duty at \$1.50 each and 15% on the former, and \$6 each and 35% on the latter?
2. Find the duty on 400 yards of Brussels carpet $\frac{3}{4}$ yard wide, valued at \$1.25 per running yard, the rate being 44 cents per square yard and 40% ad valorem.

3. Forty-six million pounds of beet sugar were imported last year. What sum was received in duties at 1.615 cents per pound?

4. What per cent of the cost is paid on pearl-handled table knives valued at \$3 per dozen, the rate being 16 cents each and 15%?

5. Find the duty at $1\frac{1}{2}\text{¢}$ each and 15%, on 100 dozen table knives valued at 50¢ per dozen.

6. An importer pays a duty of 20¢ each and 40%, on penknives costing him \$4 per dozen in England. At what price per dozen must he sell them to make a profit of 25% above the cost in this country after the duty is paid?

7. Snowshoes to the value of \$7099.65 were imported last year. What was the duty at 20%?

8. From Cuba 4128 pounds of starch were imported, the duty being $1\frac{1}{2}$ cents per pound, less 20%. How much duty was received?

9. What is the duty at 1 cent per cubic foot on 100 hewn timbers, each 16 feet long, 2 feet wide, and 1 foot thick?

10. What would be the total cost, including duty, of 100 pieces of oil cloth, each 20 yards long and 3 yards wide, at 25¢ per square yard, the duty being 8¢ per square yard and 15%?

STOCKS AND BONDS.

STOCKS.

116. When a very large amount of money is required to undertake and carry on a business enterprise, a **stock company**, or **corporation**, is generally organized.

117. The capital stock of a corporation consists of the amount invested, which is divided into *shares* of stock, generally of \$100 each. To each stockholder is issued a certificate signed by the proper officers, which specifies the number of shares he owns and the original value of the share.

The following model shows the usual wording of a stock certificate :

INCORPORATED UNDER THE LAWS OF THE STATE OF ILLINOIS	
No. 436.	150 Shares.
CENTRAL MANUFACTURING COMPANY	
This Certifies that <i>James H. Flynn</i>	
is the owner of <i>One Hundred Fifty</i> shares of	
<i>One Hundred Dollars each of the full-paid stock</i>	
<i>of the Central Manufacturing Company.</i>	
Transferable only on the books of the company in person or by attorney upon surrender of this certificate.	
<i>Frank Carroll</i> Secretary.	<i>Gerald Tofte</i> President.

118. When the enterprise shows a profit, the directors determine what portion of the profits should be retained to extend and improve the business, and distribute the remainder among the stockholders as a *dividend*. The dividend is generally fixed as a certain per cent of the capital stock. Dividends may be paid annually, semi-annually, or oftener.

119. The original value of stock named in the certificate is called the *face* or *par value*. When the corporation is

paying satisfactory dividends, the stock generally sells above par; if the profits are small or nothing, the stock sells below par.

120. Stocks are generally bought and sold through brokers, who receive a brokerage (commission) of \$12.50 for each purchase or sale of 100 shares of stocks of the par value of \$100 each, regardless of the price actually paid. This amounts to $\frac{1}{8}\%$ of the *par value*. Thus, a broker selling 100 shares of Baltimore & Ohio for \$12,200 would receive the same brokerage, \$12.50, as would be paid him for the sale of 100 shares of Erie for \$4425.

121. Oral Problems.

1. A company is incorporated with a capital of \$150,000. How many shares of \$100 each?

2. What per cent of the capital stock of \$100,000 is held by the owner of 25 shares?

NOTE. If no other value is given, the par value is assumed to be \$100.

3. If a dividend of 3% is declared, how much is received by the owner of 40 shares of stock?

4. What is the annual income of the owner of 25 shares of stock on which is paid a semi-annual dividend of 4%?

5. What is the value of 20 shares of Baltimore & Ohio selling at $122\frac{1}{2}$?

6. What must be paid for 10 shares of N.Y. Central at $119\frac{7}{8}$ and the brokerage of $\frac{1}{8}\%$?

7. If I pay \$125 for a share of stock, and receive \$5 dividends annually, what rate per cent do I receive annually on my investment?

8. I pay \$150 for a \$100 share of stock and receive in dividends 4% annually on my investment. What per cent of the par value is the dividend?

9. What per cent dividend must be declared in order that I may receive 4% on the sum I have paid for stock that cost me 250?

10. I receive a semi-annual dividend of 4% on Adams Express Co. stock for which I paid 300. What per cent do I receive annually on my investment?

Written Exercises.

122. Find the cost to the purchaser, adding $\frac{1}{8}\%$ brokerage in each case:

1. 20 shares of Delaware, Lackawanna, & Western, par value \$50 each, at 465.

$\$50 \times 20 \times 4.65\frac{1}{8} = \4651.25 . *Ans.* The par value of 20 shares is $\$50 \times 20$, or \$1000. The price, including the brokerage, is $465\frac{1}{8}\%$ of the par value. $\$1000 \times 4.65\frac{1}{8}$ gives the total cost.

To find the cost of stock, multiply the par value of the stock by the price (including the brokerage) expressed as a per cent.

NOTE. The par value is taken at \$100 per share, unless otherwise specified.

2. 100 shares of Pennsylvania R.R., \$50 each, at $119\frac{3}{8}$.

3. 60 shares of Toledo, St. Louis, & Western, at 28.

4. 100 shares of National Lead Co., at $59\frac{3}{4}$.

5. 10 shares of Canadian Pacific, at $166\frac{1}{2}$.

6. 250 shares of Iowa Central, at 40.

Find the amount received by the seller, after the deduction of the brokerage of $\frac{1}{8}\%$:

7. 72 shares of Amalgamated Copper at $84\frac{5}{8}$.

$\$100 \times 72 \times .84\frac{5}{8} = \6084 . *Ans.* The par value of 72 shares is \$7200. Price, $84\frac{5}{8}\% - \frac{1}{8} = 84\frac{1}{2}\%$, deducting brokerage. $\$7200 \times .84\frac{1}{2}$ gives the amount paid over to the seller.

8. 175 shares of Chicago & Alton, at 16.

9. 450 shares of Corn Products Refining Co., at $75\frac{1}{8}$.

10. 20 shares of Illinois Central, at 137.

123. Find the per cent realized upon an investment in American Car & Foundry Co. at $39\frac{7}{8}$, dividends $\frac{1}{2}\%$ quarterly.

Assuming that the stock is bought through a broker, its cost will be $39\frac{7}{8}\% + \frac{1}{8}\%$, or 40% of the par value. The annual dividend is 2% of the par value; that is, the owner of a one-hundred-dollar share receives \$2 annual dividend on his investment of \$40. The problem resolves itself into finding what per cent \$2 is of \$40.

$$\text{Rate} = 2\% \div .40 = 5\%. \text{ Ans.}$$

To find the rate yielded by a stock investment, divide the annual rate of dividend by the cost (including brokerage) considered as a per cent.

124. Find the rate yielded by an investment in the following:

NOTE. Add $\frac{1}{8}\%$ brokerage to the price of each.

1. Western Union Telegraph at $79\frac{7}{8}$, quarterly dividend $1\frac{1}{4}\%$.

2. Great Northern, at $124\frac{7}{8}$, quarterly dividend $1\frac{3}{4}\%$.

3. Chesapeake & Ohio at $37\frac{3}{8}$, annual dividend 1% .

4. People's Gas at $89\frac{7}{8}$, quarterly dividend $1\frac{1}{2}\%$.

5. Rock Island, at $49\frac{7}{8}$, annual dividend 1% .

125. Written Problems.

1. A broker sold for Mr. Freeman 285 shares of Delaware & Hudson at $187\frac{1}{8}$. What did Mr. Freeman receive? Find the broker's commission.

2. Canada Southern stock is bought for me at $60\frac{1}{8}$. The broker's bill is \$7230, which includes his commission of $\frac{1}{8}\%$. How many \$100 shares does he buy for me?

3. Mr. Curran received from his broker \$10,174.50 as the proceeds of Union Pacific stock sold at 134. What was the par value of the shares?

4. John Griffin paid \$9330 for 80 shares of Consolidated Gas bought through his broker. What rate did the broker pay?

5. A man bought 25 shares of Union Pacific at $148\frac{1}{4}$ and sold them at $146\frac{7}{8}$. What was his loss, including $\frac{1}{8}\%$ each for buying and for selling?

BONDS.

126. When the United States government, a state, or a city wishes to borrow money, it issues **bonds**, which are agreements to pay at a stated time a certain sum of money, and a certain percentage thereon annually, semi-annually, or quarterly, until the redemption of the bonds. The money paid for the use of the borrowed money is called *interest*.

127. In addition to sums raised by the sale of stock, many corporations obtain money by the issue of bonds. The payment of the bonds is frequently secured by providing that the holders may sell the property of the corporation if default should be made in the payment of the face value of the bonds as they become due, or in the payment of the interest instalments.

128. The holders of stock in a corporation are the owners; the holders of bonds are persons to whom the stockholders are indebted for money loaned. The stockholders receive dividends that vary according to the profits of the corporation. The bondholders receive a fixed rate on the money loaned, and the principal sum when the bonds are due. When this is paid, the bonds cease to exist; the stock continues while the corporation lasts. Stock is issued in shares of the same par value for each share; bonds are issued of different par values; say of \$100, \$500, \$1000, etc.

NOTE. Bonds are described as, for instance, Illinois Central 4's, 98 $\frac{5}{8}$; which means that the bonds of the Illinois Central Railroad paying 4% interest sell at 98 $\frac{5}{8}$.

129. Oral Exercises.

Find the annual income on the following bonds:

1. Par value \$3000; rate 5%.
2. Par value \$5000; rate 4%.
3. Par value \$4000; rate 6%.
4. Par value \$5000; rate 5%.
5. Par value \$10,000; rate 3.65%.

Find the cost of the following bonds, adding $\frac{1}{8}\%$ brokerage:

6. Par value \$3000; price 119 $\frac{7}{8}$.
7. Par value \$4000; price 87 $\frac{3}{8}$.
8. Par value \$1000; price 99 $\frac{3}{4}$.
9. Par value \$2000; price 123.
10. Par value \$50; price 84 $\frac{1}{8}$.

Find the proceeds of sales of the following bonds, less brokerage:

11. Par value \$3000; price 120 $\frac{1}{8}$.
12. Par value \$2000; price 87 $\frac{1}{8}$.
13. Par value \$1000; price 99 $\frac{3}{4}$.
14. Par value \$4000; price 125 $\frac{1}{8}$.
15. Par value \$1600; price 87 $\frac{5}{8}$.

Written Exercises.

130. 1. Find the cost of bonds of the par value of \$7000 bought at 118 $\frac{3}{4}$, brokerage $\frac{1}{8}\%$.

$$\text{Cost} = \$7000 \times (1.18\frac{3}{4} + .00\frac{1}{8}).$$

To find the cost of bonds, multiply the par value by the price (including the brokerage) expressed as a per cent.

Find the cost of the following, including brokerage:

2. \$4000 Imperial Japanese $4\frac{1}{2}$'s at $92\frac{3}{8}$.
3. \$7000 U. S. of Mexico 5's at 98.
4. \$6000 Republic of Cuba 5's at $102\frac{1}{4}$.
5. \$8000 Philippine 4's at 111.
6. \$5000 Wisconsin Central 4's at $85\frac{1}{2}$.
7. Find the amount received for \$6000 Northern Pacific 4's sold at $100\frac{3}{4}$, less brokerage at $\frac{1}{8}\%$.

$$\$6000 \times (1.00\frac{3}{4} - .00\frac{1}{8}).$$

Find the amount received for the following, less brokerage:

8. \$3500 Western Union $4\frac{1}{2}$'s, at 95.
 9. \$4500 Canada Southern 5's, at $101\frac{7}{8}$.
 10. \$2500 Kansas City Southern 3's, at 69.
 11. \$1500 Lake Shore $3\frac{1}{2}$'s, at $93\frac{3}{4}$.
 12. \$10,000 Short Line 4's, at $88\frac{3}{4}$.
- 131.** 1. Find the annual income from Albany & Susquehanna $3\frac{1}{2}$'s, par value \$7500.

$$\text{Par value} \times \text{rate of interest} = \text{annual income}$$

$$\$7500 \times .03\frac{1}{2}$$

Find the annual income of:

2. Lake Shore $3\frac{1}{2}$'s, par value \$3300.
3. Lehigh and Wilkesbarre $4\frac{1}{2}$'s, par value \$4800.
4. Baltimore and Ohio $3\frac{1}{2}$'s, par value \$1750.
5. Hocking Valley $4\frac{1}{2}$'s, par value \$12,500.
6. District of Columbia 3.65's, par value \$8000.

132. In ascertaining the *rate of income* produced by a bond bought either above or below par, consideration is taken of the fact that par value is to be paid when the bond falls due. For this reason the rate of income actually yielded may differ materially from the rate specified in the bond.

The calculation of this actual rate of income involves the application of the principles of compound interest, in a manner so complicated that it cannot be treated in a common school arithmetic.

Bankers and brokers use for this purpose carefully prepared tables, and the question of obtaining the actual rate of income yielded by a given bond should generally be referred to these tables.

Quotations as above were recently made on bonds as follows :

BONDS.	RATE.	DUE.	PRICE.	YIELDING.
City of Omaha, Nebraska,	4½	Sept. 1927	104.25	4.20 %
State of Tennessee,	4½	Oct. 1913	102.00	4.10 %
South Dakota Central Railway,	5	Jan. 1917	95.00	5.40 %
Northern Illinois Light & Traction Co.,	5	July 1923	95.00	5.50 %

COMMERCIAL DISCOUNT.

133. It is the practice of many manufacturers of certain classes of goods to issue catalogues of their products, in which they quote prices higher than those at which the goods are actually sold. The price printed in the catalogue is the *list price*. Their customers receive a *discount sheet* notifying them of the rate of discount, a new sheet being sent at each fluctuation of the actual selling price.

List prices are sometimes known as *catalogue prices*. *Commercial discounts* are also called *trade discounts*.

134. Two or more discounts are frequently given : as, 70 and 30% ; 50 and 17½% ; 60, 10, and 10%, etc. One of these discounts is deducted from the list price as a base ; the remainder is the base from which the next discount is deducted. Thus, \$ 200 less 70 and 30% would indicate that \$ 200 was reduced by 70%, or \$ 140, leaving a remainder of \$ 60, and that this remainder was to be reduced 30%, or \$ 18, leaving a net price of \$ 42.

NOTE. The mark % is generally written only after the last of a series of successive discounts.

135. It is usual in some branches of the wholesale trade to allow a discount for payment made before the expiration of the credit time. Thus, a bill of goods amounting to \$400 payable in 30 days might be reduced 2% by payment at the time of purchase, or 1% by payment at any time within 15 days. This *cash discount* is deducted from the cost remaining after the deduction of the other discounts.

136. The following bill shows the method of stating trade and cash discounts:

SLATINGTON, PENN., Jan. 16, 1908.

MR. RICHARD HADDEN

Bought of THE HUGO KATZLER COMPANY

Terms: Cash less 2%.

100 gross slates, # 14	\$ 8.75	875			
less 20%		175			
		700			
less 10%		70			
		630			
less 5%		31	50		
		598	50		
Cash, less 2%		11	97		
Received payment,				\$586	53
Jan. 16, 1908,					
The Hugo Katzler Company.					
per C. B. J. Smythe.					

137. Oral Exercises.

Find the net prices:

1. List price \$ 80; discounts 50 and 50%
2. List price \$ 100; discounts 50 and 25%
3. List price \$ 160; discounts 25 and 10%
4. List price \$ 200; discounts 10 and 10%

5. List price \$ 300; discounts $33\frac{1}{3}$ and 20%
6. List price \$ 800; discounts 50 and 10%
7. List price \$ 500; discounts 20 and 10%
8. List price \$ 400; discounts 25 and 20%
9. List price \$ 60; discounts $16\frac{2}{3}$ and 10%
10. List price \$ 50; discounts 80 and 80%

138. Written Exercises.

1. Find the net price of goods listed, or catalogued, at \$484, on which are allowed trade discounts of 25 and 10%, and a cash discount of 5%.

$$\begin{array}{r}
 \$968 \\
 \text{less } \frac{1}{4} \quad 242 \\
 \hline
 \$726 \\
 \text{less } \frac{1}{10} \quad 72.60 \\
 \hline
 \$653.40 \\
 \text{less } \frac{1}{20} \quad 32.67 \\
 \hline
 \text{Net } \$620.73
 \end{array}$$

Business men do not write unnecessary figures. 25% of \$968 is obtained by dividing it by 4. To divide 726 by 10, the numbers composing it are rewritten one place to the right. To divide by 20, divide by 2, placing each quotient figure one place to the right.

Find the net prices:

List price.	Discounts.	List price.	Discounts.
2. \$1484	50 and 10%	7. \$1800	50, 10, and 10%
3. \$371.90	40 and 30%	8. \$2400	30, 10, and 5%
4. \$225.80	60 and 20%	9. \$4000	60, 25, and 10%
5. \$3075	70 and 30%	10. \$1700	10, 10, and 5%
6. \$4500	25 and 20%	11. \$800	40, 30, and 10%

139. Written Problems.

1. What is due on a bill of hardware amounting at list prices to \$480, the discounts being 40, $12\frac{1}{2}$, and 10%?

2. Find the difference on a bill of \$700 between a single discount of 50%, and successive discounts of 25, 15, and 10%.

3. Abraham Stein paid \$330 in settlement of a bill for corrugated iron pipe amounting at list prices to \$800. One

of the two successive discounts was 50% ; what was the other?

4. On a bill of goods listed at \$ 100, compare discounts of 40, 25, and 10%, discounts of 10, 25, and 40%, and discounts of 25, 40, and 10%.

5. A buyer pays 80% of 90% of the catalogue price of \$ 875 for a piano. What does he pay for it? What two successive discounts does he receive? What per cent of the catalogue price does he pay? What equivalent single discount does he receive?

6. A man had \$ 630 after spending 20% of his money and then 10% of the remainder. What sum had he at first?

7. Make out a bill for 250 dozen penknives sold by George Freiberg to Nicholas J. Barnett at \$12 per dozen less 25 and 10%, and 5% for cash.

8. I am offered 40 and 10% by one dealer and 30 and 20% by another. Which should I accept, and what would I save on a purchase of goods listed at \$ 275?

9. A dealer marks an article 50% above \$80, its cost, and sells it at a discount of 30% from the marked price. What per cent does he gain?

10. What per cent would be lost by selling an article for \$ 90, which was 40% below the marked price, the latter being 50% above cost?

140. Preliminary Exercises.

1. A man has \$ 100. How much will he have left after spending $\frac{3}{10}$ of it and $\frac{1}{10}$ of the remainder?

2. What fraction of his money has a man after spending $\frac{3}{10}$ of it?

3. If a man has $\frac{7}{10}$ of a certain sum and spends $\frac{1}{10}$ of this $\frac{7}{10}$, what fraction of this $\frac{7}{10}$ remains?

4. What fraction of a man's money equals $\frac{9}{10}$ of $\frac{7}{10}$ of his money ?

5. What fraction of a man's money is left after he spends $\frac{3}{10}$ of it and $\frac{1}{10}$ of the remainder?

6. Find the value of $(1 - \frac{3}{10}) \times (1 - \frac{1}{10})$.

7. What per cent is $\frac{9}{10}$ of 70 % ?

8. 90 % of 70 % equals what per cent ?

9. 30 and 10 % discount equals what per cent net ?

The deduction of the first discount of 30% leaves a remainder of 70 per cent, 90% of which gives a net price of 63 per cent of the list price.

Ans. 63%.

141. Oral Exercises.

Find the per cent remaining after the deduction of the following discounts:

1. 50 and 50 %

6. 20 and 20 %

2. 40 and 40 %

7. 10 and 10 %

3. 60 and 10 %

8. 50, 20, and 10 %

4. 70 and 30 %

9. 30, 20, and $12\frac{1}{2}$ %

5. 60 and 20 %

10. $33\frac{1}{3}$, 10, and 10 %

11. What single discount equals successive discounts of 30 and 10 % ?

The first discount is 30 %

The second discount is $\frac{1}{10}$ of 70 % $\frac{7}{10}$ %

Total discount 37 %

Find the single discount equivalent to each of the following:

12. 50 and 50 %

17. 20 and 20 %

13. 40 and 40 %

18. 10 and 10 %

14. 60 and 10 %

19. 50, 20, and 10 %

15. 70 and 30 %

20. 30, 20, and $12\frac{1}{2}$ %

16. 60 and 20 %

21. $33\frac{1}{3}$, 10, and 10 %

CHAPTER III.

APPLICATIONS OF PERCENTAGE INVOLVING THE ELEMENT OF TIME.

INTEREST.

142. A person in need of money for any purpose may obtain it by furnishing proper security for its repayment at a specified time, and by agreeing, in return for the use of the money, to pay a certain per cent of the sum loaned.

143. The sum paid for the use of money is called **interest**. The sum borrowed is called the **principal**. The *rate* is the per cent agreed upon for the use of the money for a *year*. The *amount* is the principal plus the interest.

144. Oral Exercises.

Find the interest on \$800 for one year at:

- | | | |
|--------|---------------------|----------------------|
| 1. 3 % | 5. $2\frac{1}{2}$ % | 9. $4\frac{1}{4}$ % |
| 2. 4 % | 6. $3\frac{1}{4}$ % | 10. $4\frac{1}{2}$ % |
| 3. 5 % | 7. $3\frac{1}{2}$ % | 11. $4\frac{3}{4}$ % |
| 4. 6 % | 8. $3\frac{3}{4}$ % | 12. $5\frac{1}{2}$ % |

Find the yearly interest at 4 % on:

- | | | |
|-----------|-----------|-----------|
| 13. \$200 | 17. \$250 | 21. \$275 |
| 14. \$400 | 18. \$350 | 22. \$375 |
| 15. \$600 | 19. \$450 | 23. \$475 |
| 16. \$800 | 20. \$550 | 24. \$575 |

145. Find the yearly interest on :

- | | |
|-------------------------------|-------------------------------|
| 1. \$ 300 at 4% | 5. \$ 600 at $4\frac{1}{2}\%$ |
| 2. \$ 400 at 5% | 6. \$ 200 at $5\frac{1}{2}\%$ |
| 3. \$ 500 at 6% | 7. \$ 400 at $3\frac{3}{4}\%$ |
| 4. \$ 100 at $3\frac{1}{2}\%$ | 8. \$ 800 at $8\frac{1}{4}\%$ |

Find the interest at 6 % on :

- | | |
|-------------------------------------|----------------------------------|
| 9. \$ 100 for 3 years | 13. \$ 50 for 4 years |
| 10. \$ 250 for 2 years | 14. \$ 80 for $\frac{1}{2}$ year |
| 11. \$ 400 for $1\frac{1}{2}$ years | 15. \$ 70 for 6 months |
| 12. \$ 300 for $2\frac{1}{2}$ years | 16. \$ 120 for 4 months |

146. In calculating interest the general rule is to consider the year as consisting of 12 months of 30 days each.

147. General Method.

1. Find the interest on \$350 for 3 years 6 months, at 6%.

Principal	\$ 350
Rate	<u>.06</u>
Interest for 1 year	\$ 21.00
Time in years	<u>$3\frac{1}{2}$</u>
	\$ 10.50
	<u>63.00</u>
Interest for 3 yr. 6 mo.	\$ 73.50

Find the interest for a year by multiplying the principal by the rate considered as hundredths. Multiply the interest for one year by the number of years.

Find the interest on :

- \$ 1628 at 4% for 3 years
- \$ 2156 at 5% for 9 years
- \$ 2326 at 3% for 7 years

5. \$1668 at 3% for 8 years
6. \$620 at 5% for 4 years
7. \$262.50 at 3% for 6 years
8. \$346.75 at 4% for 4 years
9. \$679.40 at 5% for 6 years
10. \$548 at $5\frac{1}{2}\%$ for 2 years
11. \$2166 at $4\frac{1}{2}\%$ for 3 years
12. \$325.20 at 5% for 6 years
13. \$628.40 at 4% for 6 years
14. \$769.10 at 3% for 7 years
15. \$878.60 at 4% for 4 years
16. \$340 at $3\frac{1}{2}\%$ for 2 years 6 months
17. \$584.50 at $4\frac{1}{2}\%$ for 3 years 3 months
18. \$736.40 at $3\frac{3}{4}\%$ for 4 years 9 months
19. \$825.60 at $4\frac{1}{4}\%$ for 3 years 8 months
20. \$1243.20 at $5\frac{1}{2}\%$ for 4 years 4 months

148. Cancellation Method.

1. Find the interest on \$1875 for 2 years 7 months 21 days at 4%.

$$\begin{array}{rcl}
 2 \text{ years} & = & 720 \text{ days} \\
 7 \text{ months} & = & 210 \text{ days} \\
 21 \text{ days} & = & 21 \text{ days} \\
 2 \text{ yr. } 7 \text{ mo. } 21 \text{ da.} & = & 951 \text{ days} \\
 & = & \frac{951}{360} \text{ yr.}
 \end{array}
 \quad
 \begin{array}{r}
 \$5 \\
 \$75 \\
 \$1875 \times \frac{4}{100} \times \frac{951}{360} = \frac{\$1585}{8} = \$198.12\frac{1}{2}
 \end{array}$$

Ans. \$198.13 -

This method is frequently found convenient when the time contains days, the whole number of days being found and placed above 360, as a denominator, to reduce the time to years. The rate is generally expressed as a fraction: $4\frac{1}{2}\%$ as $\frac{9}{200}$; $3\frac{3}{4}\%$ as $\frac{15}{400}$, etc. The operations are indicated without reducing any fractions to lower terms, the shortening of the work by cancellation accomplishing the same result.

Find the interest and the amount:

2. \$1300 at 3% for 72 days
3. \$1700 at 5% for 100 days
4. \$5200 at 2% for 2 months 6 days
5. \$840 at 4% for 16 days
6. \$136 at 6% for 8 months 8 days
7. \$240 at $4\frac{1}{2}\%$ for 13 months 20 days
8. \$480 at $2\frac{1}{4}\%$ for 1 yr. 1 mo. 20 da.
9. \$360 at $3\frac{3}{4}\%$ for 2 yr. 4 mo. 15 da.
10. \$1800 at 6% for 3 yr. 7 mo. 21 da.

149. When the time between two given dates is less than a year, it is usual in calculating interest to ascertain the exact number of days between the two dates.

1. Find the interest on \$685 at 5% from July 1 to September 23.

The time remaining in the first month is found by subtracting the given date from the number of days in the month. To this are added the whole number of days in the intervening month and the number of days expressed by the date of the last month.

July 1 to July 31, 30 days	\$6.85		$\frac{7}{100}$		$\frac{84}{360}$		$= \frac{\$47.95}{6} = \$7.99 +$
August, 31 "	\$685	\times	$\frac{5}{100}$	\times	$\frac{84}{360}$	$=$	$\frac{\$47.95}{6} = \$7.99 +$
In September, 23 "					$\frac{72}{6}$		
84 days							Ans. \$7.99

NOTE. The 100 in a denominator is generally canceled by changing the decimal point in a numerator.

Find the interest and the amount:

2. \$420 at $3\frac{1}{2}\%$ from March 6 to May 2
3. \$360 at $4\frac{1}{2}\%$ from August 16 to October 23
4. \$540 at 4% from May 16 to November 13
5. \$680 at 6% from March 23 to July 7

6. \$720 at 5% from June 28 to August 16
7. \$1080 at $2\frac{1}{3}\%$ from Dec. 12, 1907, to Feb. 29, 1908
8. \$1440 at 4% from Nov. 11, 1907, to March 24, 1908
9. \$960 at 6% from Oct. 10, 1907, to Feb. 25, 1908
10. \$640 at 5% from July 17, 1907, to April 6, 1908

150. When the dates are more than a year apart, the time is found by ascertaining first the number of years, then the number of months, then the number of days.

1. Find the amount of \$765 at $4\frac{1}{2}\%$ from Jan. 17, 1908, to May 3, 1910.

The time from Jan. 17, 1908, to Jan. 17, 1910 is 2 yr., or 720 da.

The time from Jan. 17, 1910, to April 17, 1910 is 3 mo., or 90 da.

The time from April 17, 1910, to May 3, 1910 is 16 da.

The time from Jan. 17, 1908, to May 3, 1910 is 826 da.

$$\begin{array}{r} \$765 \\ \$7.65 \\ \text{Interest} = \$765 \times \frac{9}{200} \times \frac{413}{360} = \frac{\$315.945}{4} = \$78.99 - \end{array}$$

$$\text{Amount} = \$765 + \$78.99 = \$843.99. \text{ Ans.}$$

Find the amount:

2. \$300 at 6% from Dec. 27, 1907, to Oct. 20, 1909
3. \$400 at 5% from July 16, 1907, to May 10, 1910
4. \$500 at 4% from Sept. 14, 1907, to July 8, 1911
5. \$600 at $4\frac{1}{2}\%$ from Apr. 20, 1907, to Dec. 5, 1912

151. Method by Aliquot Parts.

1. Find the interest on \$247.56 at 4% for 3 years 8 months 27 days. Find the amount.

All the work necessary to obtain the result is given here. Under the principal is written the interest for a year, by multiplying by 4 and writing the first figure of the product two places to the right. The interest for the remaining 2 years is next found. The interest

for 8 months is obtained by dividing by 3 the interest for 2 years. The interest for 24 days is $\frac{1}{10}$ of the interest for 8 months, or 240 days. The interest for 3 days is $\frac{1}{8}$ of the interest for 24 days. Adding together the interest items, we get the interest. By including the principal, we obtain the amount.

Principal	<u>\$ 247.56</u>	
Interest for 1 yr.	\$ 9.9024	
Interest for 2 yr.	19.8048	
Interest for 8 mo.	6.6016	$\frac{1}{3}$ of int. for 2 yr.
Interest for 24 da.	.6602	$\frac{1}{10}$ of int. for 8 mo.
Interest for 3 da.	<u>.0823</u>	$\frac{1}{8}$ of int. for 24 da.
Interest for 3 yr. 8 mo. 27 da.	\$ 37.05 +	Ans.
Amount	\$ 284.61 +	Ans.

2. Find the amount of \$ 247.56 at $4\frac{1}{2}\%$ for 3 years 8 months 27 days.

The interest for 2 years is first found, employing 9 as a multiplier instead of $4\frac{1}{2}$, which would be used to obtain the interest for 1 year. Aliquot parts, other than those used in the previous example, are employed to show that variety is possible.

Principal	\$ 247.56	
Interest for 2 yr. }	22.2804	
Interest for 1 yr. }	11.1402	$\frac{1}{2}$ of 2 yr.
Interest for 6 mo. }	5.5701	$\frac{1}{2}$ of 1 yr.
Interest for 2 mo. }	1.8567	$\frac{1}{3}$ of 6 mo.
Interest for 18 da. }	.5570	$\frac{1}{10}$ of 6 mo.
Interest for 9 da. }	<u>.2785</u>	$\frac{1}{2}$ of 18 da.
Amount	\$ 289.24 +	Ans.

3. Find the interest on \$125 at 6% for 1 yr. 11 mo. 14 da.

Interest for 1 year	\$ 7.50	
Interest for 6 months		$\frac{1}{2}$ of \$ 7.50
Interest for 4 months		$\frac{1}{3}$ of \$ 7.50
Interest for 1 month		$\frac{1}{4}$ of int. for 4 mo.
Interest for 12 days		$\frac{1}{10}$ of int. for 4 mo.
Interest for 2 days		$\frac{1}{6}$ of int. for 12 da.

4. Find the amount of \$240 at 5% for 3 yr. 5 mo. 18 da.

Principal	\$240.
Interest for 2 yr.	24.
Interest for 1 yr.	
Interest for 4 mo.	
Interest for 1 mo.	
Interest for 15 da.	
Interest for 3 da.	

5. Find the amount of \$720 at $5\frac{1}{2}\%$ for 4 yr. 7 mo. 21 da.

Principal	\$720.	
Interest for 2 yr.	79.20	11% of principal
Interest for 2 yr.	79.20	
Interest for 6 mo.		$\frac{1}{4}$ of int. for 2 yr.
etc.		etc.

6. Find the interest on \$124.75 at 7% for 2 yr. 6 mo. 18 da.

1 yr., 1 yr., 6 mo., 18 da.

152. When the time is less than a year, and the rate is 6%, advantage is taken of the fact that the interest for 60 days is 1% of the principal.

1. Find the amount of \$248 at 6% for 208 days.

Principal	\$248.	
Interest for 60 da.	2.48	1% of principal
Interest for 120 da.	4.96	twice int. for 60 da.
Interest for 20 da.	.8267	$\frac{1}{3}$ of int. for 60 da.
Interest for 6 da.		$\frac{1}{10}$ of int. for 60 da.
Interest for 2 da.		$\frac{1}{3}$ of int. for 6 da.

2. Find the interest on \$1256 at 6% for 66 days.

60 da. + 6 da.

Find the interest at 6% :

- | | |
|---------------------------|---------------------------|
| 3. \$1873 for 84 days. | 6. \$249 for 219 days. |
| 4. \$956.50 for 167 days. | 7. \$157.40 for 175 days. |
| 5. \$48.76 for 315 days. | 8. \$863.20 for 93 days. |

Find the amount, the rate being 6% :

9. \$1200 from Feb. 7, 1909, to Aug. 15, 1909.
10. \$1200 from Mar. 8, 1908, to Sept. 14, 1908.
11. \$1200 from Jan. 9, 1907, to Dec. 1, 1907.
12. \$1200 from Apr. 15, 1908, to June 26, 1908.
13. \$1200 from May 25, 1909, to Nov. 19, 1909.
14. \$1200 from July 30, 1908, to Oct. 29, 1908.

153. Interest is the product of three factors : the principal, the rate, and the time. By the ordinary method the interest is calculated for a year, and this result is multiplied by the number of years ; that is,

Interest for given time = interest for 1 year \times number of years.

154. Some examples are more readily worked by multiplying the principal by the rate for the given time ; that is,

Interest for given time = principal \times rate for given time.

155. Preliminary Exercises.

Find the rate :

1. For 60 days, at 6% per year.
2. For 4 years, at 5%.
3. For 2 years, at $4\frac{1}{2}\%$.
4. For 4 years, at $3\frac{3}{4}\%$.
5. For 1 month, at 6%.
6. For $\frac{1}{3}$ year, at 5%.
7. For 72 days, at 5%.
8. For 216 days, at 5%.
9. For 144 days, at 5%.
10. For 288 days, at 5%.
11. For 36 days, at 5%.
12. For $\frac{1}{3}$ year, at $4\frac{1}{2}\%$.
13. For 40 days, at $4\frac{1}{2}\%$.
14. For 80 days, at $4\frac{1}{2}\%$.
15. For 160 days, at $4\frac{1}{2}\%$.

156. Sight Exercises.

Find the interest:

- | | | |
|-----------------------------------|--------------------------|------------------|
| 1. Rate 6% ; | time 60 days; | principal \$ 146 |
| 1% of the principal. | | |
| 2. Rate 5% ; | time 2 years; | principal \$ 430 |
| $\frac{1}{10}$ of the principal. | | |
| 3. Rate $4\frac{1}{2}\%$; | time 2 years; | principal \$ 109 |
| 9% of the principal. | | |
| 4. Rate $3\frac{3}{4}\%$; | time 4 years; | principal \$ 100 |
| 15% of the principal. | | |
| 5. Rate 6% ; | time 1 month; | principal \$ 184 |
| $\frac{1}{2}\%$ of the principal. | | |
| 6. Rate 5% ; | time $\frac{1}{5}$ year; | principal \$ 173 |
| 7. Rate 5% ; | time 72 days; | principal \$ 217 |
| 8. Rate 5% ; | time 216 days; | principal \$ 203 |
| 9. Rate 5% ; | time 144 days; | principal \$ 324 |
| 10. Rate 5% ; | time 288 days; | principal \$ 112 |
| 11. Rate 5% ; | time 36 days; | principal \$ 408 |
| 12. Rate $4\frac{1}{2}\%$; | time $\frac{1}{9}$ year; | principal \$ 246 |
| 13. Rate $4\frac{1}{2}\%$; | time 40 days; | principal \$ 444 |
| 14. Rate $4\frac{1}{2}\%$; | time 80 days; | principal \$ 681 |
| 15. Rate $4\frac{1}{2}\%$; | time 160 days; | principal \$ 312 |
| 16. Rate 6% ; | time 2 months; | principal \$ 517 |
| 17. Rate 6% ; | time 4 months; | principal \$ 408 |
| 18. Rate 6% ; | time 180 days; | principal \$ 123 |
| 19. Rate 6% ; | time 240 days; | principal \$ 204 |
| 20. Rate 6% ; | time 90 days; | principal \$ 222 |

157. Six Per Cent Method.

Find the interest on \$384.75 for 3 years 7 months 23 days, at 6%.

The rate for 3 years at .06 per year is .18. The rate for one month is $\frac{1}{12}$ of .06, or $.00\frac{1}{2}$, which gives .035 as the rate for 7 months. The rate for one day is $\frac{1}{30}$ of $.00\frac{1}{2}$, or $.000\frac{1}{6}$, which gives $.003\frac{5}{6}$ as the rate for 23 days. The total, $.218\frac{5}{6}$, is the rate for 3 years 7 months 23 days at .06 per year.

The principal, \$384.75, is then multiplied by the rate for the given time, $.218\frac{5}{6}$.

Ans. \$84.20

$$\begin{array}{r}
 \$384.75 \\
 .218\frac{5}{6} \\
 \hline
 6)1.92375 \\
 .32063 \\
 3.07800 \\
 3.8475 \\
 76.950 \\
 \hline
 \$84.19613
 \end{array}$$

The interest at 6% may be calculated by multiplying the principal by the rate for the given time, obtained by adding together .06 times the number of years, $.00\frac{1}{2}$ times the number of months, and $.000\frac{1}{6}$ times the number of days.

158. Written Exercises.

Find the interest at 6% on the following:

1. \$1206 for 1 yr. 6 mo. 24 da.
2. \$375.30 for 2 yr. 8 mo. 18 da.
3. \$854.10 for 1 yr. 1 mo. 6 da.
4. \$579.60 for 3 yr. 7 mo. 12 da.
5. \$840 for 4 yr. 3 mo. 13 da.

159. Find the interest on \$384.75 for 3 years 7 months 23 days: (a) at 5%; (b) at $4\frac{1}{2}\%$; (c) at 3%; (d) at $3\frac{3}{4}\%$; (e) at 4%.

In Art. 157, the interest on the foregoing principal for the given time was found to be \$84.196 +.

$$\begin{array}{ll}
 \text{Interest at } 6\% = \$84.196 & \\
 \text{Deduct int. at } 1\% = 14.033 & 1\% = \frac{1}{6} \text{ of } 6\% \\
 (a) \quad \text{Interest at } 5\% = \$70.16 + \text{Ans.} & \\
 \text{Interest at } 6\% = \$84.196 & \\
 \text{Deduct int. at } 1\frac{1}{2}\% = 21.049 & 1\frac{1}{2}\% = \frac{1}{4} \text{ of } 6\% \\
 (b) \quad \text{Interest at } 4\frac{1}{2}\% = \$63.15 - \text{Ans.} &
 \end{array}$$

$$2)\$84.196 = \text{Interest at } 6\%$$

$$(c) \text{ Interest at } 3\% = 42.10 - \text{Ans.}$$

$$\text{Interest at } 6\% = \$84.196$$

$$\left\{ \begin{array}{l} \text{Interest at } 3\% = \$42.098 \\ \text{Interest at } \frac{3}{4}\% = 10.524 \end{array} \right.$$

$$3\% = \frac{1}{2} \text{ of } 6\%$$

$$\frac{3}{4}\% = \frac{1}{4} \text{ of } 3\%$$

$$(d) \text{ Interest at } 3\frac{3}{4}\% = \$52.62 + \text{Ans.}$$

$$\text{Interest at } 6\% = \$84.196$$

$$\text{Deduct interest at } 2\% = 28.065$$

$$2\% = \frac{1}{3} \text{ of } 6\%$$

$$(e) \text{ Interest at } 4\% = \$56.13 + \text{Ans.}$$

160. Find the interest on \$354.60 for 3 years 9 months 17 days. Find the amount at each rate.

- | | | | |
|------------------------|------------------------|------------------------|------------------------|
| 1. At 6 % | 3. At 7 % | 5. At 5 % | 7. At $4\frac{1}{4}\%$ |
| 2. At $3\frac{1}{2}\%$ | 4. At $3\frac{3}{4}\%$ | 6. At $4\frac{1}{2}\%$ | 8. At $5\frac{1}{2}\%$ |

NOTE. The pupil will observe that while the *interest* at 3% is one half the interest at 6%, the same is not true as to the respective *amounts*.

Find the amount:

- Principal \$1742; rate 4%; time 1 yr. 3 mo. 20 da.
- Principal \$1260; rate 7%; time 2 yr. 4 mo. 6 da.
- Principal \$484; rate 5%; time 3 yr. 6 mo. 9 da.
- Principal \$216; rate $3\frac{1}{2}\%$; time 4 yr. 10. mo. 12 da.
- Principal \$350; rate $4\frac{1}{2}\%$; time 5 yr. 3 mo. 5 da.

EXACT INTEREST.

161. In all government transactions, interest is calculated on the basis of a year of 365 days. This is called **exact interest**.

Business banks, in computing interest due them, employ the year of 360 days, but they pay out interest on the basis of the 365-day year.

162. Written Exercises.

Find the exact interest:

- On \$346.50 for 187 days at 6%.

$$\$346.50 \times \frac{6}{100} \times \frac{187}{365}$$

2. On \$73 for 92 days at 4%.
3. On \$147 at 5% from March 1 to May 13.
4. On \$284 at 3.65% from June 12 to Sept. 3.
5. On \$109.50 at $4\frac{1}{2}\%$ for 27 days.

INTEREST-BEARING NOTES.

163. A **promissory note** is a written promise to pay a specified sum of money. The person signing the note is the *maker*; the *payee* is the person to whom the note is made payable.

164. 1.

$\$250\frac{00}{100}$ Chicago, Ill. Jan. 16, 1908
 Ten months after date I promise to pay to
 the order of William S. Hurley
 Two Hundred Fifty $\frac{00}{100}$ Dollars
 Value received with interest at six per cent.
 Herman A. Metz.

Find the amount of the above note at maturity, Nov. 16, 305 days.

2.

MARION, IND., Dec. 27, 1907.

On demand after date I promise to pay to Clement Marsh, or order, Three Hundred Seventy-five $\frac{00}{100}$ Dollars, value received, with interest at five per cent.

$\$375\frac{00}{100}$

LOUIS HAFF.

How much was due on this note at settlement, April 4, 1908?

3. A note for \$360, dated July 20, 1907, was paid with interest March 18, 1908. Find the amount.

4. What sum will pay a note for \$1080 drawn Sept. 25, and due Dec. 31, with interest at $5\frac{1}{2}\%$?

5. Find the amount of a note for \$760 dated Oct. 29, 1907, and paid Feb. 29, 1908, with interest at 6 per cent.

PARTIAL PAYMENTS.

165. Merchants' Rule.

GRAND RAPIDS, WIS., Oct. 10, 1907.

On demand after date I promise to pay to the order of Thomas May Two Hundred Seventy Dollars, value received, with interest at 5 per cent.

\$ 270 $\frac{00}{100}$

NATHAN S. JOHNS.

On this note the following payments were made :

Nov. 16, 1907; \$ 50.

Dec. 30, 1907; \$ 100.

Jan. 18, 1908; \$ 75.

Find the balance due at settlement, March 29, 1908.

Face of note		\$ 270.
Interest on \$ 270 Oct. 10 to March 29, 171 days		6.41
	Amount	<u>\$ 276.41</u>
Payment Nov. 16, 1907	\$ 50.	
Interest Nov. 17 to March 29, 134 days	.93	
Payment Dec. 30, 1907	100.	
Interest Dec. 30 to Mar. 29, 90 days	1.25	
Payment Jan. 18, 1908	75.	
Interest Jan. 18 to Mar. 29, 71 days	.74	227.92
Balance due		<u>\$ 48.49</u>

When partial payments are made on an interest-bearing note, the sum due at settlement, if made within a year after the note is drawn, is found by the following method :

Find the amount of the note to the time of settlement, and from it deduct the sum of the amounts of the payments, the interest on each of the latter being calculated from the date of each payment to the date of settlement.

NOTE. When a partial payment is made, it is generally written on the back of a note. For this reason the payment is said to be *indorsed*.

166. Find the sum due at settlement of the following notes:

1. Drawn Jan. 9, 1907; face \$ 500; rate 5 %; settled Dec. 18, 1907. Payment of \$ 300, July 16, 1907.

2. Drawn June 15, 1909; face \$ 1000; rate 6 %; settled Jan. 7, 1910. Payments:

Aug. 12, 1910; \$ 300.

Dec. 15, 1910; \$ 500.

3. Drawn Jan. 8, 1908; face \$ 600; rate 6 %; settled Dec. 8, 1908. Payments:

April 8, 1908; \$ 200.

July 8, 1908; \$ 200.

Oct. 8, 1908; \$ 200.

4. Drawn July 5, 1909; face \$ 1500; rate 5 %; settled June 16, 1910. Payments:

Sept. 8, 1909; \$ 200.

Nov. 3, 1909; \$ 300.

Feb. 10, 1910; \$ 400.

Apr. 9, 1910; \$ 500.

167. United States Rule.

FLINT, MICH., Nov. 5, 1906.

On demand, I promise to pay to George J. Gilmartin, or order, Eight Hundred Seventy $\frac{00}{100}$ Dollars, value received, with interest at six per cent.

SAMUEL B. DONALDSON.

\$ 870 $\frac{00}{100}$.

On this note are indorsed the following payments; Dec. 5, 1907, \$ 50; May 4, 1909, \$ 300; Dec. 6, 1909, \$ 300. What sum is due at settlement, Nov. 5, 1910?

Face of note		\$ 870.
Interest on \$ 870 from Nov. 5, 1906, to Dec. 5, 1907, 1 yr. 1 mo.		56.55
As the payment of \$ 50 is less than the interest, the interest on \$ 870 is computed from Dec. 5, 1907, to May 4, 1909, 1 yr. 4 mo. 29 da.		73.805
Amount May 4, 1909		<u>\$ 1000.355</u>
Less payment Dec. 5, 1907	\$ 50	
Less payment May 4, 1909	300	350.
Due May 4, 1909		<u>\$ 650.355</u>
Interest from May 4, 1909, to Dec. 6, 1909, 216 da.		23.412
Amount Dec. 6, 1909		<u>\$ 673.767</u>
Payment Dec. 6, 1909		300.
Due Dec. 6, 1909		<u>\$ 373.767</u>
Interest from Dec. 6, 1909, to Nov. 5, 1910, 334 da.		20.806
Due Nov. 5, 1910		<u>\$ 394.573</u>
		Ans. \$ 394.57

Find the amount of the principal to the time when a payment, or the sum of two or more payments, is at least equal to the interest then due. From this amount subtract the payment or payments.

Consider the remainder as a new principal and proceed as before.

168. As there is no requirement in the foregoing note that interest is due at any specified time, it would be unfair to the maker of the note to add \$56.35 interest and deduct the first payment of \$50, thus requiring him to pay interest for a time on \$876.35, a sum greater than the face of the note. The rule that interest shall not be required on interest prevails in most of the states.

169. In these and all other examples in interest, the time exceeding a year is found in years, months, and days; time less than a year is found by taking the exact number of days. The year is considered as having 360 days, except in the examples specifying exact interest.

170. Find the amount due at settlement:

1. Face of note \$200; rate 6%; drawn Jan. 15, 1908; date of settlement July 20, 1910. Payment \$100 May 5, 1909.

2. Face of note \$ 600 ; rate 6% ; drawn April 6, 1906 ; date of settlement Nov. 1, 1910. Payments, \$ 50. Jan. 7, 1908 ; \$ 250 May 1, 1909.

3. Face of note \$ 800 ; rate 6% ; drawn Aug. 21, 1906 ; date of settlement Aug. 21, 1910. Payments, \$ 200 Sept. 21, 1907 ; \$ 200 Oct. 21, 1908 ; \$ 200 Nov. 24, 1909.

4. Face of note \$ 1200 ; rate 6% ; drawn Nov. 13, 1905 ; settled Nov. 13, 1909. Payments, \$ 60 Nov. 13, 1906 ; \$ 500 April 19, 1907 ; \$ 50 May 23, 1907 ; \$ 500 June 18, 1908.

COMPOUND INTEREST.

171. A deposit of \$ 100 is made in a savings bank July 1, 1907. On Jan. 1, 1908, the depositor is entitled to $\frac{1}{2}$ year's interest at 4% per annum, or \$2. If he does not withdraw the interest, it is credited to his account, and July 1, 1908, he is entitled to interest on \$102, or \$2.04. If this is not withdrawn, he is entitled on Jan. 1, 1909, to $\frac{1}{2}$ year's interest on \$ 104.04, which is \$ 2.08.

172. **Compound interest** is interest taken at regular periods upon the principal and unpaid interest.

173. Written Exercises.

1. Find the compound interest on \$ 600 for 3 years 6 months at 6%.

Principal,	\$ 600.00
Interest, 1st year,	<u>36.00</u>
Amount, 1st year,	\$ 636.00
Interest, 2d year,	<u>38.16</u>
Amount, 2d year,	\$ 674.16
Interest, 3d year,	<u>40.4496</u>
Amount, 3d year,	\$ 714.6096
Interest, 6 months,	<u>21.4383</u>
Amount, 3 yr. 6 mo.,	\$ 736.05
Principal,	<u>600.00</u>

Compound Interest, 3 yr. 6 mo., \$ 136.05 Ans.

2. Find the interest on \$600 for 3 years 6 months at 6%, compounded semi-annually.

Principal,	\$ 600.00
Interest 6 months,	<u>18.00</u>
Amount 6 months,	\$ 618.00
Interest 6 months,	<u>18.54</u>
Amount, 1 year,	\$ 636.54
etc.	etc.

To find the compound interest, calculate the amount for the first period. Using this as a new principal, calculate the amount for the next period, continuing in this way until the end of the last period. The difference between the last amount and the principal will be the compound interest.

3. Find the amount of \$800 for 4 years at 5%, compound interest.

4. What is the difference between the simple interest of \$1000 at 4% for 3 years and interest at the same rate compounded quarterly?

5. Find the amount of \$800 for 4 years at 5% compounded semi-annually.

Principal,	\$ 800.00
Interest for 6 mo. $\frac{1}{4}\%$	<u>20.00</u>

Divide by 4, writing the first quotient figure one place to the right.

6. How much less will be the interest on \$800 for 4 years at 5%, when compounded semi-annually than when compounded quarterly?

ANNUAL INTEREST.

174.

ALPENA, MICH., July 1, 1906.

On demand after date I promise to pay to the order of Thomas Tierney, Four Hundred Dollars, value received, with interest at six per cent payable annually.

\$ 400 $\frac{00}{100}$

JOHN J. BARNICLE.

By the terms of this note an interest payment of \$ 24 was due July 1, 1907. In the case of non-payment at that date, the laws of some states permit the collection of *simple interest* on the interest due until it is paid.

175. Annual interest is simple interest on the principal and upon each deferred interest payment for the time each remains unpaid.

176. Find the amount due on the foregoing note on July 1, 1911, by annual interest, provided no payments are made.

	Principal, \$ 400.
Interest July 1, 1906 to July 1, 1911 ; 5 years at 6%	120.
Interest July 1, 1907, to July 1, 1911 ; 4 years at 6% on \$ 24	5.76
Interest July 1, 1908, to July 1, 1911 ; 3 years at 6% on \$ 24	
Interest July 1, 1909, to July 1, 1911 ; 2 years at 6% on \$ 24	
Interest July 1, 1910, to July 1, 1911 ; 1 year at 6% on \$ 24	_____
Amount due July 1, 1911, \$	

It will be noted that there are several items of interest on \$ 24, making an aggregate of 10 years' interest on \$ 24.

To find the amount due on a note bearing annual interest, add to the amount of the principal at simple interest, the simple interest on each unpaid annual interest for the time it remained unpaid.

177. Written Exercises.

1. What is the difference between the simple interest on \$ 400 at 6% for 5 years, and the compound interest? Between the simple interest and the annual interest? Between the annual interest and the compound interest?

2. Find the annual interest on a note for \$ 1000 at 5% in 6 years, no payment of any kind having been made.

3. What is due after 6 years on a note for \$ 1000 bearing 6% annual interest, the first three interest payments having been made?

4. Robert P. Webb gives a note for \$1000 dated July 1, 1906, agreeing to pay annual interest at 6%. He pays \$500 July 1, 1908. Find the amount due July 1, 1910.

PRESENT WORTH.

178. If I owe a debt of \$106 payable in a year, and money is worth 6%, a present payment of \$100 should cancel the indebtedness, since the latter sum will amount in a year, at 6%, to \$106.

179. The *present worth* of \$106 payable in a year is \$100 when the prevailing rate of interest is 6%. The *true discount* in this case is \$106 — \$100, or \$6.

180. The **present worth** of a given sum is the sum that, if placed at interest at the given rate, will amount to the former sum at the time the former sum is payable.

181. *To find the present worth of a given sum, payable at a future date, divide the given sum by the amount of one dollar for the given time at the given rate.*

182. *To find the true discount, subtract the present worth from the sum payable at the future date.*

183. Problems in present worth are interesting rather than practical. Persons to whom money is payable at a future date, as a rule, agree to accept the sum at once less a discount of a given per cent, which is frequently much larger than the prevailing rate of interest.

184. Written Exercises.

1. What sum placed at interest for 9 months at 6% will amount to \$376.20?

2. Find the true discount on \$376.20 paid 9 months before it is due, money being worth 6%.

3. What is the bank discount of \$376.20 for 9 months at 6% ?

4. I owe \$2500 payable in 1 yr. 6 mo. What sum invested at 4% will amount to that sum when it is due ?

5. Find the present worth and the true discount of \$2500 due in 2 years, money being worth 6%.

BANKS AND BANKING.

185. Banks are of two general kinds, which may be distinguished as **business banks** and **savings banks**.

186. The ordinary business bank, or **bank of discount and deposit**, is a stock company organized according to law with a paid-up capital not less than a certain specified sum. The bank is required to keep on hand at all times a certain percentage of the total sum due depositors to meet their demands.

Deposited in			
THE UNION NATIONAL BANK			
by			
<u>John Doe</u>			
Racine, Wis., <u>June 11</u> , 190 <u>8</u>			
	DOLLARS	CENTS	
Bills	350		
Specie	610		
Checks, enter separately	14	75	
	53	80	
	1 028	55	

187. A person depositing money fills out a deposit slip in the accompanying form and receives a bank book in which the deposit is entered.

188. The depositor's book is ruled as shown in the accompanying illustration. On the left are entered the deposits as they are made; on the opposite page are entered in double columns the sums paid to the depositor or to others on his written order.

THE UNION NATIONAL BANK of RACINE, WISCONSIN, Dr. in account with					John Doe Cr.				
1908			1028	55		16	55		
	Dep.					43	10		
June 11						120			

189. A depositor withdraws money from his bank by means of a **check**. The illustration on the opposite page shows one form of a check book; the portion on the right, constituting the check, is detached; and the other, called the *stub*, is retained in the book as a memorandum for the depositor of the amounts withdrawn.

190. The average business bank pays no interest on moneys deposited, except by special arrangement. The bank takes care of the money of its depositors, enables them to settle their bills by sending the amounts due through the mails in the form of checks, and loans money to them on approved security.

191. A **savings bank** is organized to take charge of small sums. To encourage thrift in the community, small deposits are solicited, sums of one dollar, or less, being

received. Interest is paid or credited to the depositors at stated periods.

192. A bank book is given the depositor, as in the case of the business bank. Withdrawals of money are not made by check; the depositor, as a rule, presents himself with his bank book, in which the sum withdrawn is entered at the time. The interest to which the depositor is entitled is also entered in his bank book.

BANK DISCOUNT.

193. In addition to loaning money on real estate or other security, banks lend money for short periods to responsible customers on the personal security of the borrower, the repayment of the money when due being, as a rule, guaranteed by another reliable person.

194. John A. Wilder desires to borrow \$1500 from his bank for 90 days. Frank H. Hartridge is wil-

Brought forward	\$1628.55	Carried forward	1650
June 17, 1908 order of J. Smith for insurance			
\$1628.55 No. 1		Pay to the order of John Smith Sixteen hundred and 50/100 Dollars John Doe	
THE UNION NATIONAL BANK Racine, Wis., June 17, 1908 No. 1		\$1628.55	

ling to become security for the repayment of the loan. Mr. Wilder makes out his note in the following form:

\$1500 ⁰⁰ / ₁₀₀	Philadelphia, Pa.,	Dec. 4, 1907
Ninety days after date I promise		
to pay to the order of Frank H. Hartridge		
Fifteen Hundred ⁰⁰ / ₁₀₀	Dollars	
at the Deshler Bank.		Value received.
John A. Wilder		

Frank H. Hartridge *indorses* the note by writing his name on the back, thereby agreeing to pay the face of the note when due, in case Mr. Wilder fails to meet his obligation. Mr. Hartridge's indorsement also transfers the note to the bearer, the Deshler National Bank, if the bank authorities advance the money.

The bank discounts the note by handing over to Mr. Wilder, or placing to his credit, \$1477.50; that is, \$1500 less \$22.50, for 90 days' interest.

195. The *face of the note* is \$1500; the interest deducted in advance, \$22.50, is called the *bank discount*; the balance paid or credited to Mr. Wilder, \$1477.50, is called the *proceeds*. The note is due 90 days after Dec. 4, 1907, or March 3, 1908, which is called the *day of maturity*. The number of days from the time of discounting the note to the day of maturity is called the *term of discount*.

196. In a few states the law grants the maker of a note three days, called *days of grace*, in addition to the time specified in the note. Interest is charged for these days, as well as for any other days allowed by law. In some

states and cities in which Saturday is a half holiday, a note regularly falling due on Saturday is not payable until Monday; in others, a note falling due on a holiday is payable the next previous business day.

NOTE. In the examples in bank discount no notice will be taken of holidays or days of grace.

197. In discounting a note, the bank deducts interest for the period between the date of discount and the date of maturity. Notes intended for discount are generally presented the day they are drawn, but in some cases the holder of the note does not present it for discount at once. In this case the *term of discount* is shorter than the time for which the note is drawn.

198. Notes offered for discount are generally drawn for 30, 60, or 90 days; or for 1, 2, 3, or 4 months. A 30-days note drawn Feb. 1, 1909, is due 30 days thereafter, or March 3; a 1-month note is due March 1.

* **Preliminary Exercises.**

199. Find the date of maturity of the following notes:

1. 30-days note drawn Jan. 15.
2. 1-month note drawn Jan. 15.
3. 60-days note drawn Feb. 1.
4. 2-months note drawn Feb. 1.
5. 90-days note drawn June 1.
6. 3-months note drawn June 1.

200. Find the term of discount:

1. Discounted Feb. 1, due March 1.
2. Discounted March 3, due April 15.
3. Discounted Feb. 29, due May 5.
4. Discounted Nov. 1, due Dec. 30.
5. Discounted June 25, due Aug. 4.
6. Discounted Sept. 1, due Nov. 1.

TO FIND THE BANK DISCOUNT.**Oral Exercises.****201.** Find the bank discount:

NOTE. The bank discount is the interest for the term. When no rate is specified, 6% is understood.

1. Term 60 days; face \$ 475.
2. Term 40 days; face \$ 300.
3. Term 90 days; face \$ 100.
4. Term 15 days; face \$ 160.
5. Term 32 days; face \$ 600.
6. Term 18 days; face \$ 240.

202. Find the proceeds:

NOTE. To find the proceeds, deduct the discount from the face.

1. Face \$ 475; term 60 days.
2. Face \$ 300; term 40 days.
3. Face \$ 100; term 90 days.
4. Face \$ 160; term 15 days.
5. Face \$ 600; term 32 days.
6. Face \$ 240; term 18 days.

203. Find the bank discount on a note for \$ 600:

1. Due March 1, discounted Feb. 2.
2. Due April 15, discounted March 3.
3. Due May 5, discounted Feb. 29.
4. Due Dec. 30, discounted Nov. 1.
5. Due Aug. 4, discounted June 25.
6. Due Nov. 1, discounted Sept. 1.

204. Find the date of maturity, the term of discount, the bank discount, and the proceeds:

Face \$ 157.80; drawn June 8, 1908; time 3 months; discounted July 1; rate $5\frac{1}{2}\%$.

Date of maturity — 3 months after June 8, or Sept. 8. *Ans.*

Term of discount — July 1 to Sept. 8, 69 days. *Ans.*

Interest at 6% on \$ 157.80 for 60 days = \$ 1.578

6 days = .1578

3 days = .0789

Bank discount at 6% \$ 1.8147

Deduct $\frac{1}{12}$ for discount at $\frac{1}{2}\%$.1512

Bank discount at $5\frac{1}{2}\%$ \$ 1.66 + *Ans.*

Proceeds, \$ 157.80 — \$ 1.66 = \$ 156.14 *Ans.*

NOTE. In states allowing days of grace, the date of maturity is Sept. 11, the term of discount is 72 days, the bank discount is \$ 1.74, and the proceeds \$ 156.06.

To find the bank discount on a note, find the interest on the face of the note for the term of discount.

205. Written Exercises.

Find the date of maturity, the term of discount, the bank discount, and the proceeds:

1. Face \$ 346.50; drawn June 8, 1909; time 90 days; discounted July 1, 1909; rate 6%.

Date of maturity, 90 days after June 8 — Sept. 6;

Term of discount — 67 days.

2. Face \$ 540; drawn Feb. 4, 1908; time 30 days; discounted Feb. 4; rate 6%.

3. Face \$ 1224; drawn Jan. 6, 1908; time 60 days; discounted Jan. 16, 1908; rate 6%.

4. Face \$ 874.50; drawn Nov. 30, 1909; time 3 months; discounted Nov. 30, 1909; rate 6%.

Due Feb. 28, 1910.

5. Face \$ 376.20; drawn Aug. 4, 1907; time 30 days; discounted Aug. 14, 1907; rate 5%.

6. Face \$4000; drawn Feb. 29, 1908; time 90 days; discounted March 30, 1908; rate $5\frac{1}{2}\%$.

7. Face \$573.30; drawn Dec. 8, 1910; time 4 months; discounted Dec. 8, 1910; rate 6% .

8. Face \$1872; drawn March 9, 1909; time 60 days; discounted April 29, 1909; rate $4\frac{1}{2}\%$.

9. Face \$351.90; drawn Oct. 20, 1908; time 2 months; discounted Oct. 20, 1908; rate 6% .

DISCOUNT OF INTEREST-BEARING NOTES.

206. In settlement of his account Mr. Niver gives the following note to Sullivan, McDonald, & Co.

BUFFALO, N.Y., *May 4, 1908.*

Four months after date I promise to pay to the order of *Sullivan, McDonald, & Co., Seven Hundred Eighty-four $\frac{75}{100}$* Dollars, value received, at the Marine Bank, with interest at six per cent.

$\$784\frac{75}{100}$

Harmon B. Niver.

This note is discounted at 6% by the Marine Bank the day it is drawn. Find the proceeds.

Date of maturity = May 4 + 4 months, or Sept. 4

Term (27 + 30 + 31 + 31 + 4) days, or 123 days

Face of note — \$784.75

Add	{	Interest for 60 days	7.85-
		Interest for 60 days	7.85-
		Interest for 3 days	.39

Amount due Sept. 4 \$800.84

Deduct	{ Discount for 60 days	\$ 8.01	}
	{ Discount for 60 days	8.01	
	{ Discount for 3 days	<u>.40</u>	
Proceeds, May 4		\$784.42	

NOTE. It will be observed that the bank discount, \$16.42, exceeds the interest, \$16.09, by 33 cents. This difference of 33 cents is the interest for 123 days on \$16.09, the interest on the face of the note.

To find the bank discount on an interest-bearing note, find the interest for the term of discount on the amount of the note at maturity.

207. In a non-interest-bearing note, the discount is taken on the sum given in the face of the note. In an interest-bearing note, the sum due at maturity is the *amount* of the note; that is, the face of the note plus the interest for the time stated in the note.

208. Written Exercises.

Find the proceeds of the following interest-bearing notes, the rate of interest in each case being 6%, and the rate of discount 6%:

1. Sixty-days note for \$1872, drawn March 9, 1909, discounted April 29, 1909.

2. Ninety-days note for \$4000, drawn Feb. 29, 1908, discounted March 30, 1908.

3. Thirty-days note for \$376.20, drawn Aug. 4, 1907, discounted Aug. 14, 1907.

4. Sixty-days note for \$1224, drawn Jan. 6, 1908, discounted Jan. 16, 1908.

5. Three-months note for \$157.80, drawn June 8, 1908, discounted July 1, 1908.

PROBLEMS IN INTEREST AND BANK DISCOUNT.

TO FIND THE TIME.

209. Preliminary Exercises.

1. Find the interest on \$200 for 3 years at 5%.
2. The interest on \$200 at 5% is \$30. Find the time.
3. The interest on \$200 for 3 years is \$30. Find the rate.
4. The interest for 3 years at 5% is \$30. Find the principal.
5. Interest \$36; time 3 years; rate 6%. Find the principal.

Written Exercises.

210. Find the time in which a principal of \$240 at 6% will yield interest as follows:

- | | | |
|------------|------------|-----------|
| 1. \$14.40 | 4. \$18 | 7. \$1.80 |
| 2. \$1.44 | 5. \$19.80 | 8. \$0.72 |
| 3. \$15.84 | 6. \$21.60 | 9. \$0.04 |

211. Find the time in which a principal of \$240 at 6% will produce amounts as follows:

- | | | |
|-------------|-------------|-------------|
| 1. \$254.40 | 4. \$259 | 7. \$245.76 |
| 2. \$241.44 | 5. \$260.80 | 8. \$240.68 |
| 3. \$255.84 | 6. \$262.60 | 9. \$240.08 |

212. In what time will \$300 at 6% yield \$49.95 interest?

Representing time by T , we have

$$300 \times \frac{6}{100} \times T = 49.95$$

or

$$18 T = 49.95$$

$$T = 49.95 \div 18$$

The time in years is $\frac{1}{18}$ of 49.95. Dividing 49.95 years by 18 gives a quotient of 2 years and a remainder of 13.95 years. This is reduced to 167.4 months. Dividing by 18, gives a quotient of 9 months and a remainder of 5.4 months, or 162 days. Dividing 162 days by 18 gives a quotient of 9 days.

$$\begin{array}{r}
 18 \overline{)49.95} \text{ years (2 years} \\
 \underline{36.} \\
 \text{remainder } 13.95 \text{ years} \\
 \text{or 13.95 times 12 months} \\
 18 \overline{)167.4} \text{ months (9 months} \\
 \underline{162} \\
 \text{remainder } 5.4 \text{ months} \\
 \text{or 5.4 times 30 days} \\
 18 \overline{)162.0} \text{ days (9 days} \\
 \underline{162}
 \end{array}$$

Ans. 2 yr. 9 mo. 9 da.

An examination of the foregoing will show that the time in years is found by dividing the interest, \$49.95, by the interest on \$300 at 6% for 1 year, which is \$18.

That is, if \$18 interest is produced in a year, the number of years required to produce \$49.55 interest will be the quotient of 49.55 by 18.

To find the time in years, divide the given interest by the interest for one year on the given principal at the given rate.

213. When the *amount* is given, the interest is found by deducting the principal from the amount.

The foregoing problem might read: In what time will \$300 at 6% amount to \$349.95?

In this case the interest is found to be \$349.95 - \$300, or \$49.95, after which the problem is solved as shown above.

PROOF:

Principal \$300	
6%	18.00 interest for 1 year
6%	18.00 interest for 1 year
$\frac{1}{2}$ year	9.00 interest for 6 months
$\frac{1}{2}$ of 6 months	4.50 interest for 3 months
$\frac{1}{10}$ of 3 months	.45 interest for 9 days
<hr/>	
\$349.95 amount for 2 yr. 9 mo. 9 da.	

214. Find the time:

1. Principal \$240; rate 6%; interest \$14.40.
2. Principal \$250; rate 5%; amount \$262.50.
3. Principal \$300; rate 4%; interest \$14.50.
4. Principal \$300; rate 4%; amount \$315.50.
5. Principal \$800; rate $4\frac{1}{2}\%$; interest \$124.40.

TO FIND THE TERM OF DISCOUNT.

215. Oral Exercises.

1. Principal \$300; rate 6%; interest \$9. Time in years? In months? In days?
2. Face of note \$300; rate 6%; bank discount \$9. Term in months? In days?
3. Face of note \$300; rate 6%; bank discount \$4.50. Term in months?
4. Face of note \$300; rate 6%; bank discount \$0.45. Term in days?
5. Face of note \$600; rate 6%; bank discount \$6. Term in days?

Written Exercises.

216. Find the number of days in which a note of \$240, discounted at 6%, will yield discounts as follows:

- | | | |
|-----------|-----------|-----------|
| 1. \$3.60 | 4. \$6.68 | 7. \$2.92 |
| 2. \$0.36 | 5. \$4.32 | 8. \$3.48 |
| 3. \$3.96 | 6. \$1.24 | 9. \$1.76 |

217. Find the number of days in which a note of \$240, discounted at 6%, will yield proceeds as follows:

- | | | |
|-------------|-------------|-------------|
| 1. \$236.40 | 4. \$238.76 | 7. \$235.68 |
| 2. \$239.64 | 5. \$238.24 | 8. \$233.32 |
| 3. \$236.04 | 6. \$236.52 | 9. \$237.08 |

218. In what time will the bank discount on a note for \$300 be \$13.95?

As the term of discount is generally required in days, the discount, \$13.95, is divided by the discount (interest) for 1 day on \$300 at 6%.

The interest on \$300 for a year is \$18; for a day, it is $\frac{1}{360}$ of \$18, or 5¢. The time in days = $13.95 \div .05$, or 279. *Ans.* 279 days.

To find the term of discount in days, divide the given discount by the discount for one day at the given rate on the given sum.

219. Find the term of discount:

1. Face of note \$240; rate 6%; bank discount \$3.76.
2. Face of note \$840; rate 5%; proceeds \$835.10.
3. Face of note \$1260; rate 6%; bank discount \$13.86.
4. Face of note \$1320; rate 6%; proceeds \$1311.20.
5. Face of note \$1500; rate 5%; bank discount \$18.75.

TO FIND THE RATE OF INTEREST.

Preliminary Exercises.

220. Find the rate at which \$200 will yield yearly interest as follows:

- | | | |
|---------|---------|---------|
| 1. \$10 | 4. \$9 | 7. \$6 |
| 2. \$13 | 5. \$12 | 8. \$11 |
| 3. \$7 | 6. \$8 | 9. \$14 |

221. Find the rate at which \$200 will yield interest as follows:

- | | |
|---------------------|---------------------|
| 1. \$10 in 1 year. | 4. \$2 in 60 days. |
| 2. \$5 in 6 months. | 5. \$3 in 3 months. |
| 3. \$18 in 2 years. | 6. \$30 in 3 years. |

222. Find the rate at which \$ 200 will produce the following amounts :

- | | |
|-----------------------|--------------------------|
| 1. \$ 210 in 1 year. | 4. \$ 215 in 2 yr. 6 mo. |
| 2. \$ 228 in 2 years. | 5. \$ 202 in 60 days. |
| 3. \$ 218 in 2 years. | 6. \$ 204 in 90 days. |

Written Exercises.

223. Find the rate at which \$ 240 will produce yearly interest as follows :

- | | | |
|-------------|-------------|-------------|
| 1. \$ 12 | 3. \$ 13.20 | 5. \$ 16.80 |
| 2. \$ 10.80 | 4. \$ 14.40 | 6. \$ 15.60 |

224. Find the rate at which \$ 240 will yield interest as follows :

- | | |
|-------------------------|-------------------------|
| 1. \$ 12 in 1 year. | 4. \$ 2.80 in 60 days. |
| 2. \$ 6 in 6 months. | 5. \$ 3.60 in 3 months. |
| 3. \$ 31.20 in 2 years. | 6. \$ 32.40 in 3 years. |

225. Find the rate at which \$ 240 will produce amounts as follows :

- | | |
|---------------------------|---------------------------|
| 1. \$ 255.60 in 1 year. | 4. \$ 242.80 in 60 days. |
| 2. \$ 244.20 in 6 months. | 5. \$ 243.30 in 3 months. |
| 3. \$ 261.60 in 2 years. | 6. \$ 283.20 in 3 years. |

226. At what rate will \$ 300 yield \$ 49.95 interest in 2 years 9 months 9 days?

Representing the rate by R , we have

$$300 \times \frac{R}{100} \times \frac{999}{360} = 49.95$$

$$\frac{333 R}{40} = 49.95$$

$$R = 49.95 \div \frac{333}{40}$$

Inverting the divisor, and canceling, $\frac{49.95 \times 40}{333} = 6$. *Ans.* 6%.

An examination of the foregoing will show that the required rate is obtained by dividing the interest, \$49.95, by the interest on \$300 at 1 % for the given time, which is \$8.325, or $\frac{\$333}{40}$;

That is, if $\frac{\$333}{40}$ interest is produced in the given time at 1 %, the rate required to produce \$49.95 will be the quotient of 49.95 by $\frac{333}{40}$.

To find the rate of interest, divide the given interest by the interest at one per cent for the given principal for the given time.

227. When the *amount* is given, the interest is found by deducting the principal from the amount. (See Art. 143.)

228. Find the rate of interest :

1. Principal \$200; time 2 yr. 6 mo.; int. \$30.
2. Principal \$300; time 60 days; amt. \$303.50.
3. Principal \$400; time 1 yr. 1 mo. 1 da.; int. \$19.55.
4. Principal \$500; time 3 yr. 20 da.; amt. \$568.75.
5. Principal \$600; time 4 yr. 3 mo. 6 da.; int. \$166.40.

TO FIND THE RATE OF DISCOUNT.

229. The discount for 60 days on a note for \$120 is \$1; what is the rate?

This problem resolves itself into the following problem in interest :
At what rate will \$120 yield \$1 interest in 60 days?

230. When the *proceeds* are given, the discount is obtained by deducting the proceeds from the face of the note.

231. Find the rate of discount :

1. Face of note \$100; term 135 days; discount \$1.50.

2. Face of note \$150; term 66 days; proceeds \$148.35.

3. Face of note \$200; term 144 days; discount \$2.

4. Face of note \$250; term 72 days; proceeds \$247.

5. Face of note \$300; term 93 days; discount \$3.10.

TO FIND THE PRINCIPAL.

Preliminary Exercises.

232. Find the principal that will yield yearly interest as follows at 6%:

- | | | |
|-----------|-----------|---------|
| 1. \$6 | 4. \$15 | 7. \$27 |
| 2. \$7.20 | 5. \$7.50 | 8. \$12 |
| 3. \$18 | 6. \$21 | 9. \$33 |

233. Find the principal that will yield interest as follows at 6%:

- | | |
|----------------------------------|------------------------|
| 1. \$12 in 2 years. | 4. \$7.50 in 6 months. |
| 2. \$14.40 in 2 years. | 5. \$18 in 3 years. |
| 3. \$27 in $1\frac{1}{2}$ years. | 6. \$10 in 60 days. |

234. Find the principal that will produce the following amounts at 6%:

- | | |
|---------------------|----------------------|
| 1. \$106 in 1 year. | 4. \$224 in 2 years. |
| 2. \$424 in 1 year. | 5. \$424 in 1 year. |
| 3. \$318 in 1 year. | 6. \$118 in 3 years. |

Written Exercises.

235. Find the principal that will yield yearly interest as follows at 6%:

- | | | |
|------------|------------|-------------|
| 1. \$92.34 | 4. \$18.36 | 7. \$143.04 |
| 2. \$47.16 | 5. \$27.72 | 8. \$216.30 |
| 3. \$33.12 | 6. \$51.30 | 9. \$403.20 |

236. Find the principal that will yield interest as follows at 5% :

- | | |
|------------------------|------------------------|
| 1. \$92.34 in 1 year. | 4. \$18.36 in 4 years. |
| 2. \$47.16 in 2 years. | 5. \$27.72 in 5 years. |
| 3. \$33.12 in 3 years. | 6. \$51.30 in 6 years. |

237. Find the principal that will produce amounts as follows at 4% :

- | | |
|-------------------------|-------------------------|
| 1. \$114.40 in 1 year. | 4. \$150.80 in 4 years. |
| 2. \$162 in 2 years. | 5. \$117 in 5 years. |
| 3. \$134.40 in 3 years. | 6. \$186 in 6 years. |

238. What principal at 6 % will yield \$49.95 interest in 2 years 9 months 9 days ?

Representing the principal by P , we have

$$P \times \frac{6}{100} \times \frac{999}{360} = 49.95$$

$$\frac{333}{2000} \times P = 49.95$$

$$P = 49.95 \div \frac{333}{2000}$$

Inverting the divisor, and canceling, $\frac{49.95 \times 2000}{333} = 300$. *Ans.* \$300.

An examination of the foregoing will show that the principal is found by dividing the interest, \$49.95, by the interest on \$1 for 2 years 9 months 9 days, at 6%, which is \$.1665, or $\frac{\$333}{2000}$.

That is, if \$.1665 is produced in a given time by \$1 principal, the principal required to produce \$49.95 will be the quotient of \$49.95 by .1665.

To find the principal, divide the given interest by the interest on one dollar at the given rate for the given time.

239. What principal at 6% will amount to \$349.95 in 2 years 9 months 9 days ?

In Art. 238, the interest = $\frac{333 P}{2000}$

The amount, therefore, = $P + \frac{333 P}{2000} = \frac{2333 P}{2000}$

Then $\frac{2333 P}{2000} = 349.95$

$$P = 349.95 \div \frac{2333}{2000}$$

Inverting the divisor, and canceling, the value of P is found to be 300. *Ans.* \$300.

To find the principal, divide the given amount by the amount of one dollar at the given rate for the given time.

240. Find the principal:

1. Interest \$38.40; time 1 yr. 6 mo.; rate 4%.
2. Amount \$133.50; time 2 yr. 3 mo.; rate 5%.
3. Interest \$50.40; time 4 mo. 20 da.; rate 6%.
4. Amount \$183.60; time 5 mo. 10 da.; rate $4\frac{1}{2}\%$.
5. Interest \$49.50; time 128 days; rate $5\frac{1}{2}\%$.

TO FIND THE FACE OF A NOTE.

241. The discount at 6% for 60 days on a note is \$1; what is the face of the note?

This problem resolves itself into the following problem in interest:
What principal at 6% will yield in 60 days \$1 interest?

242. Written Exercises.

Find the face:

1. Discount \$9.84; term 60 days; rate 6%.
2. Discount \$24.30; term 90 days; rate 6%.
3. Discount \$18.92; term 120 days; rate 6%.
4. Discount \$7.87; term 66 days; rate 6%.
5. Discount \$7.32; term 72 days; rate 6%.

243. The proceeds of a note discounted for 60 days at 6% are \$99; what is the face of the note?

Let F represent face of note,

$$\text{Then } F \times \frac{6}{100} \times \frac{60}{360} = \text{discount} = \frac{F}{100}$$

$$\text{Proceeds} = F - \frac{F}{100} = \frac{99 F}{100}$$

$$\frac{99 F}{100} = 99$$

Clearing of fractions, $99 F = 9900$

$$F = 100$$

Ans. \$100.

To find the face of a note, divide the given proceeds (discount) by the proceeds (discount) of one dollar at the given rate for the given term.

244. Find the face:

1. Discount \$15.07; term 66 days; rate 6%.
2. Proceeds \$1354.93; term 72 days; rate $5\frac{1}{2}\%$.
3. Discount \$47.50; term 95 days; rate 6%.
4. Proceeds \$296.35; term 73 days; rate 6%.
5. Discount \$11; term 120 days; rate $5\frac{1}{2}\%$.

CHAPTER IV.

BUSINESS FORMS AND USAGES; REVIEW.

TRANSMISSION OF MONEY.

245. If Mr. Calkins in Omaha desires to send \$ 25 to Mr. Shaw in San Francisco, he may do so in any one of several ways. He may send his check; a *postal money order* payable in San Francisco can be procured in Omaha and sent to Mr. Shaw by mail; an express company will supply an *express money order* payable in San Francisco.

246. Large sums of money are generally transmitted through banks. The following is a *cashier's check* bought in Boston by J. C. Stewart, and sent to W. H. Ingraham, Chicago, in settlement of account.

		No. 874.
CENTRAL NATIONAL BANK,		
BOSTON, MASS., Aug. 13, 1908.		
Pay to the order of W. H. Ingraham		\$2000 $\frac{00}{100}$
Two Thousand $\frac{00}{100}$		Dollars.
To The First National Bank, }	Olis H. Luke,	
Chicago, Ill. }	Cashier.	

247. Mr. Ingraham might collect from Mr. Stewart the amount of the latter's indebtedness by means of a *sight draft*, as follows:

\$2000 $\frac{00}{100}$ CHICAGO, ILL., *Aug. 13, 1908.*

At sight pay to the order of

THE CENTRAL NATIONAL BANK OF BOSTON

Two Thousand $\frac{00}{100}$ ~~~~~~ Dollars.

Value received, and charge to account of

To *J. C. Stewart,**120 Boylston St.,**Boston, Mass.**W. H. Ingraham.*

248. Mr. Ingraham deposits this draft in his bank in Chicago, The First National, which sends it for collection to The Central National Bank of Boston. The Chicago bank charges Mr. Ingraham something for the expense of collection.

249. In the foregoing draft, W. H. Ingraham is called the *drawer*; J. C. Stewart, the *drawee*; and The Central National Bank, the *payee*.

250. The following is a form of draft employed to collect a bill due at a later date:

\$1500 $\frac{00}{100}$ BROKEN BOW, NEB., *June 16, 1908.*At *sixty days'* sight pay to the order ofTHE STOCKMEN'S NATIONAL BANK
OF FORT BENTON, MONT.*Fifteen Hundred* $\frac{00}{100}$ ~~~~~~ Dollars.

Value received, and charge to account of

To *George D. Hamlet,**Fort Benton, Mont.**Thos. M. De Laney.*

251. This is called a *time draft*. It is presented to Mr. Hamlet on June 19. If he is willing to pay it, he writes in red ink on the face of the draft the word "Accepted," followed by the date and his signature. The draft then is called an *acceptance*. It has practically become a promissory note drawn by George D. Hamlet, June 19, 1908, payable 60 days thereafter to the Stockmen's National Bank.

252. The term **exchange** is used to designate transactions by which accounts at a distance are settled without transmitting actual cash. A bank draft sometimes costs more than the sum named in the face; exchange is then at a **premium**. Under some circumstances a bank draft may be bought for less than its face; exchange is then at a **discount**. The newspapers publish the prevailing *rates of exchange*, the discount or premium given being the rate for each \$1000.

BANK DRAFTS.

253. Written Exercises.

1. What will be the cost of a draft on New York for \$12,350, purchased in Chicago at 15¢ premium?

$$\begin{array}{r}
 \text{Face of draft, } \$12,350 \\
 \text{Premium, } \$.15 \times 12.35, \quad \underline{1.8525} \\
 \$12,351.8525 \quad \text{Ans. } \$12,351.85.
 \end{array}$$

To find the cost of a draft, add the premium to the face value, or subtract the discount.

2. Find the cost of a draft for \$2346.50 at \$1 discount.

3. Mr. Jones drew on Mr. Cottier through a bank, for \$4500. How much did Mr. Jones receive if the cost of collecting the draft was $\frac{3}{10}\%$?

4. The cost of a draft for \$25,000 was \$25,015. What was the rate of exchange?

5. Mr. Bussey made a draft on Mr. Mills for \$24,000 at 60 days' sight. Find the proceeds of the draft if it is discounted at 6% by Mr. Bussey's bank, the term of discount being taken at 66 days to include the time taken to send the draft to its destination and to receive the money.

FOREIGN EXCHANGE.

254. The term *foreign bill of exchange* is generally applied to bank drafts payable in a foreign country.

255. A person who buys foreign exchange receives as a rule two bills, called a *set*. When either of the set is paid, the other becomes valueless. In some cases both bills are mailed to the payee by different vessels, and the first to reach him is presented for payment.

256. The following two bills constitute a "set" of exchange :

1 *Exchange for £1000. New York, Sept. 1, 1908.*

At sight of this first of exchange, second unpaid, pay to the order of Joseph Noonan, One Thousand Pounds Sterling, value received, and charge to account of

JOSEPH F. LAMORELLE & CO.

To Johnson & Chambers, London, Eng.

2 *Exchange for £1000. New York, Sept. 1, 1908.*

At sight of this second of exchange, first unpaid, pay to the order of Joseph Noonan, One Thousand Pounds Sterling, value received, and charge to account of

JOSEPH F. LAMORELLE & CO.

To Johnson & Chambers, London, Eng.

257. A bill of exchange may be drawn for any number of days *after sight* ; but bankers' time bills are generally for 60 days.

258. Bills of exchange differ from ordinary drafts only in being payable in a foreign currency and being frequently made out in sets of two bills.

259. The equivalent values of the units of coinage of the leading European countries are shown in the following table :

£ 1	= \$ 4.8665
1 franc	= \$ 0.193
1 mark	= \$ 0.238

The *franc* is the name of the unit in France, Belgium, and Switzerland. The *lira* in Italy, the *drachma* in Greece, the *peseta* in Spain, and the *mark* in Finland have the same intrinsic value as the franc, 19.3 cents in United States money.

The *mark* is the unit of the coinage of Germany, and the *pound sterling* that of England.

For the value of the coins of other countries, see the Appendix.

260. *Rates of foreign exchange* are given in the daily papers. The following prices were recently quoted :

	STERLING.	FRANC.	MARK.
Cable,	4.89 $\frac{7}{8}$	5.14 $\frac{3}{4}$.95 $\frac{7}{8}$
Demand,	4.88	5.16 $\frac{1}{4}$.95 $\frac{1}{4}$
60-days,	4.84 $\frac{1}{2}$	5.19 $\frac{3}{8}$.94 $\frac{1}{2}$

261. These rates show under the head of "Sterling" the cost of a pound sterling in United States dollars. For transfer by cable the rate is \$4.89 $\frac{7}{8}$ per £. A sight bill of exchange, payable on presentation in London, costs \$4.88 per £, while a bill payable 60 days after presentation costs \$4.84 $\frac{1}{2}$ per £.

262. The quotations under "francs" indicate the amount of exchange in francs that can be bought for \$1; that, for instance, \$1000 will be the cost of a cable transfer of 5147.50 francs to Paris; that the same sum will purchase a sight bill for 5162.50 francs or a 60-day bill for 5193.75 francs.

263. The quotations under "marks" give the cost of 4 marks in United States money; viz., that a cable transfer of 4000 marks costs \$958.75, a sight bill of exchange for the same sum costs \$952.50, and a 60-day bill, \$945.

264. English Money.

$$\begin{array}{rcl} 4 \text{ farthings} & = & 1 \text{ penny (d.)} \\ 12 \text{ pence} & = & 1 \text{ shilling (s.)} \\ 20 \text{ shillings} & = & 1 \text{ pound (£)} \end{array}$$

Farthings are generally written as fractions of a penny.

French Money.

$$100 \text{ centimes} = 1 \text{ franc (fr.)}$$

German Money.

$$100 \text{ pfennigs} = 1 \text{ mark (M.)}$$

265. Small sums are transmitted to foreign places by means of postal and express *money orders*.

266. Written Exercises.

1. Find the cost in Boston of a 60-day bill on Liverpool for £165 13s. 8d. at 4.85 $\frac{3}{4}$.

To find the cost of the bill we can multiply the cost of 1 £, \$4.8575, by the number of pounds.

Since 1 shilling contains 12 pence, 13s. 8d. = 164d. There are 240 pence in a pound sterling; 13s. 8d., therefore, equal $\frac{164}{240}$ of a pound, or $\frac{41}{60}$.

$$\text{The cost of the bill} = \$4.8575 \times 165\frac{41}{60}.$$

NOTE. Business men prefer to use aliquot parts.

£ 100 =	\$ 485.75	
50 =	242.875	$\frac{1}{2}$ of £ 100
10 =	48.575	$\frac{1}{10}$ of £ 100
5 =	24.2875	$\frac{1}{10}$ of £ 50
10s. =	2.4288	$\frac{1}{10}$ of £ 5
2s. =	.4858	$\frac{1}{5}$ of 10s.
1s. =	.2429	$\frac{1}{2}$ of 2s.
8d. =	.1619	$\frac{1}{3}$ of 2s.
<hr/>		
£ 165 13s. 8d. =	\$ 804.8069	
Ans.	\$ 804.81.	

2. Find the cost of £ 1000 at $4.86\frac{7}{8}$.

3. Find the cost of £ 1787 10s. at 4.85 .

4. Mr. Imlay desires to transmit by cable £ 375 9s. 6d.
Find the cost at $4.87\frac{1}{4}$.

5. Find the cost of a 60-day bill on Paris for 2160.75 fr.
at $5.18\frac{3}{4}$.

At $5.18\frac{3}{4}$ francs for \$1, the *number* of dollars will be $2160.75 \div 5.1875$.

6. A merchant owes 3000 marks in Berlin. What will
be the cost of a cable transfer at $94\frac{1}{2}$?

The cost per 4 marks being $94\frac{1}{2}$ cents, 1 mark costs $\frac{1}{4}$ of $94\frac{1}{2}$ cents,
and 3000 marks cost $(\frac{1}{4} \text{ of } 94\frac{1}{2} \text{¢}) \times 3000$.

7. I bought goods in Germany to the amount of 4520
marks. How much must I pay in New York for a bill of
exchange at $95\frac{1}{8}$ to settle the account?

8. Find the cost in Chicago of a bill on Hamburg for
10,500 marks at $95\frac{1}{4}$.

9. When the rate of exchange on Berlin is $96\frac{1}{2}$, what is
the face of a bill that costs \$ 1808.41?

10. An exporter sold to a merchant in Havre goods
amounting to \$ 3600, and drew on him at the rate of $5.20\frac{1}{4}$.
What was the face of the bill in francs?

11. A man paid \$ 1000 for a cable transfer to Dublin at 4.88. What sum was paid in Dublin ?

12. A banker paid \$10,000 for a cable transfer to Paris, in which city a bill of exchange on London was purchased with the proceeds at 25.155 francs per £. What was the face of the latter bill ?

13. Find the cost of a cable transfer to London of £ 2046 6s. 3d. at 4.88.

BILLS AND ACCOUNTS.

BILL FOR GOODS BOUGHT AT ONE TIME.

267.

LOUISVILLE, Ky., Feb. 29, 1908.

Mr. HUNTER COLLINS

Bought of KELLY & WARREN.

25 lb. Sugar	.05½	1	38	
1 bbl. Flour		5	75	
10 lb. Bacon	.12½			
2 lb. Tea	.45			
Received payment, Mch. 6, 1908,				\$
Kelly & Warren,				
per S.M.Y.				

This is a bill for purchases made Feb. 29, 1908. It was paid March 6, 1908, to the clerk who writes his initials under the name of the firm.

1. Copy the foregoing bill, filling in the missing amounts.

2. Make out a bill for the following articles sold to-day by M. L. Hutchinson to F. Curtis: 12½ yd. Dress Goods, @ 40¢; 5 yd. Ribbon, @ 63¢; 10 yd. Silk, @ 75¢; 1 Hat, \$8.75. J. H. Bancroft receipts the bill.

BILL FOR GOODS BOUGHT AT DIFFERENT TIMES.

268

MARIETTA, O., July 1, 1908.

W. S. GOODNOUGH & Co.

Sold to L. H. GULICK.

1908							
June	1	5 M Flooring	32.00				
"	10	2½ M Lath	5.00				
"	10	3 kegs Nails	2.50				
"	12	4 M Scantling	24.00				
"	20	2 M Joists	20.00				
"	20	1½ M Scantling	24.00				
"	20	2 M Lath	5.15			\$	
"	30	By Cash				25	00
"		Balance due				\$	

1. Complete the foregoing bill. Receipt it for L. H. Gulick, adding your initials as clerk.

2. Make out a bill of several items bought at a grocery during the present month.

BILL FOR SERVICES RENDERED AND MATERIAL SUPPLIED.

269.

BISMARCK, N.D., May 5, 1908.

Mr. JAMES P. HENRY

To A. S. CASWELL, Dr.

	To 5 Roses	.25				
	" 25 Geraniums	.10				
	" 3 Maples	.75				
	" 40 Pansies	.05				
	" Labor, Fertilizer, etc.		2	75		
					\$	

NOTE. Bills for goods purchased may employ the heading "Bought of," "Sold to," or the one used above. The heading here given is the appropriate one for a bill including services rendered.

1. Copy and complete the foregoing bill.

2. James H. Tully presents a bill to you, dated to-day, containing the following items: 15 days cutting wood, @ \$1.75; 6 days hauling ice, @ \$2.50; 10 days shelling corn, @ \$1.50. Make out his bill.

STATEMENT OF ACCOUNT.

270.

TOMBSTONE, ARIZ., Dec. 16, 1907.

Mr. A. L. JESSUP

In Account with FRANK R. RIX.

1907		<i>Dr.</i>					
Nov.	4	To 3 M Brick	7.00	21	00		
"	4	" 6 bbl. Cement	2.50	15	00		
"	18	" 4 bbl. Lime	1.25	5	00		
"	25	" 4 M Laths	4.25	17	00		
"	25	" 3 bbl. Lime	1.25	3	75		
"	25	" 1 bbl. Cement		2	50	\$64	25
		<i>Cr.</i>					
Nov.	11	By 1 ton Hay		14	00		
"	29	" 24 lb. Butter	.25	6	00	20	00
		Balance due				\$44	25

LEDGER ACCOUNT.

271. The foregoing statement of account is taken from the *ledger* of Frank R. Rix. Credit transactions are generally noted in a *day book*. From this book they are transferred to the ledger, in which all the accounts of an individual are brought together.

272. The following shows the ledger page containing Mr. Jessup's account:

Dr.

A. L. JESSUP.

Cr.

1907					1907			
Nov.	4	To 3 M Brick	21		Nov.	11	By 1 ton Hay	14
"	4	" 6 bbl. Cement	15		"	29	" 24 lb. Butter	6
"	18	" 4 bbl. Lime	5					
"	25	" 4 M Laths	17					
"	25	" 3 bbl. Lime	3	75				
"	25	" 1 bbl. Cement	2	50				
			64	25	Nov.	30	By Balance	44 25
								64 25
Dec.	1	To Balance	44	25				

273. In the ledger, all charges *against* the person appear on the left, and are called *debts*; the sums he *pays* or the goods he *supplies* are entered on the right, and are called *credits*. The difference between the totals of the two columns shows the amount due. Since the debit total is the greater, Mr. Jessup owes Mr. Rix the difference.

The account is *balanced* at regular intervals by adding each column and ascertaining the difference, which is \$ 44.25 in this case. This is written in red ink in the column having the smaller total, and is written in black ink in the opposite column. Mr. Jessup on Dec. 1, 1907, owes Mr. Rix \$ 44.25, which is the first entry for the month.

RECEIPTS.

274.

ALBUQUERQUE, N.M., May 25, 1908.

Received of LEE F. HANMER

One Hundred Forty-three $\frac{75}{100}$ Dollars

in full of account to date.

\$ 143 $\frac{75}{100}$.

M. E. WILLIAMS.

Receipts are all of the same general form. The one given above is a receipt "in full," showing that Mr. Hanmer has settled all his indebtedness to Mr. Williams to May 25, 1908. If Mr. Hanmer owed Mr. Williams more than \$143.75, the fourth line of the latter's receipt would read "on account," which indicates that there is an unpaid balance.

A rent receipt is of the same form, except as to the fourth line, which should read "in full of rent of premises, No. ——— St., to May 31, 1908."

MISCELLANEOUS DRILLS.

275. Sight Exercises.

Multiply:

1. 28×99 .

28 hundred — 28.

2. $28 \times 99\frac{1}{2}$.

28 hundred — 14.

3. $28 \times 99\frac{3}{4}$.

28 hundred — 7.

4. 28×24 .

700 — 28.

5. $28 \times 24\frac{1}{2}$.

700 — 14.

6. $28 \times 24\frac{3}{4}$.

700 — 7.

7. 28×50

14. $88 \times 37\frac{1}{2}$

21. $88 \times 9\frac{1}{2}$

8. 28×49

15. 88×37

22. $24 \times 19\frac{1}{2}$

9. $28 \times 49\frac{1}{2}$

16. $88 \times 36\frac{1}{2}$

23. $22 \times 39\frac{1}{2}$

10. $28 \times 49\frac{3}{4}$

17. $88 \times 37\frac{1}{4}$

24. $48 \times 24\frac{1}{2}$

11. 28×75

18. $88 \times 87\frac{1}{2}$

25. 88×125

12. $28 \times 74\frac{1}{2}$

19. $88 \times 86\frac{1}{2}$

26. 88×375

13. $28 \times 74\frac{3}{4}$

20. $88 \times 11\frac{1}{2}$

27. 88×625

276. Written Exercises.

NOTE. Do not write unnecessary figures.

1. 486×31

1458

15066 Ans.

2. 486×13

1458

6318 Ans.

$$\begin{array}{r}
 3. \quad 486 \times 313 \\
 \quad 1458 \\
 \quad 1458 \\
 \hline
 152118 \text{ Ans.}
 \end{array}$$

$$\begin{array}{r}
 4. \quad 1875 \times 99 \\
 \quad 187500 \\
 \hline
 185625 \text{ Ans.}
 \end{array}$$

In obtaining the foregoing results a line has been saved by omitting to write the multiplicand.

- | | |
|----------------------|----------------------|
| 5. 243×71 | 11. 579×213 |
| 6. 243×17 | 12. 579×312 |
| 7. 864×18 | 13. 385×99 |
| 8. 864×81 | 14. 385×999 |
| 9. 579×321 | 15. 472×999 |
| 10. 579×123 | 16. 857×99 |

$$\begin{array}{r}
 17. \quad \quad \quad 276 \\
 \quad \quad \quad \times 243 \\
 \hline
 \quad \quad \quad 828 \text{ product by 3 units} \\
 828 \times 8 \text{ tens } 6624 \text{ product by 24 tens} \\
 \hline
 \text{Ans. } 67068 \text{ product by 243}
 \end{array}$$

$$\begin{array}{r}
 18. \quad \quad \quad 276 \\
 \quad \quad \quad \times 324 \\
 \hline
 \quad \quad \quad 828 \text{ product by 3 hundreds} \\
 828 \times 8 \text{ } 6624 \text{ product by 24 units} \\
 \hline
 \text{Ans. } 89424 \text{ product by 324}
 \end{array}$$

Since 24 is 8 times 3, we can obtain the results in the foregoing examples by employing only two partial products.

- | | |
|----------------------|----------------------|
| 19. 415×426 | 23. 275×459 |
| 20. 873×642 | 24. 727×273 |
| 21. 594×742 | 25. 813×954 |
| 22. 986×427 | 26. 586×459 |

$$\begin{array}{r}
 27. \quad 157 \\
 \times 3\frac{3}{8} \\
 \hline
 471 \quad \text{product by } 3 \\
 471 \div 8 \quad 58\frac{7}{8} \quad \text{product by } \frac{3}{8} \\
 \hline
 \text{Ans. } 529\frac{7}{8} \quad \text{product by } 3\frac{3}{8}
 \end{array}$$

$$\begin{array}{r}
 28. \quad 492 \\
 \times 4\frac{4}{5} \\
 \hline
 1968 \quad \text{product by } 4 \\
 1968 \div 5 \quad 393\frac{3}{5} \quad \text{product by } \frac{4}{5} \\
 \hline
 \text{Ans. } 2361\frac{3}{5} \quad \text{product by } 4\frac{4}{5}
 \end{array}$$

$$29. \quad 265 \times 3\frac{3}{8}$$

$$32. \quad 256 \times 6\frac{6}{11}$$

$$30. \quad 976 \times 4\frac{4}{5}$$

$$33. \quad 525 \times 7\frac{7}{12}$$

$$31. \quad 151 \times 5\frac{5}{9}$$

$$34. \quad 548 \times 8\frac{8}{9}$$

$$\begin{array}{r}
 35. \quad 889 \\
 \times 24\frac{1}{2} \\
 \hline
 22225 \quad \text{product by } 25 \\
 \text{Deduct } 444\frac{1}{2} \quad \text{product by } \frac{1}{2} \\
 \hline
 \text{Ans. } 21780\frac{1}{2} \quad \text{product by } 24\frac{1}{2}
 \end{array}$$

$$\begin{array}{r}
 36. \quad 983 \\
 \times 19\frac{7}{8} \\
 \hline
 19660 \quad \text{product by } 20 \\
 \text{Deduct } 122\frac{7}{8} \quad \text{product by } \frac{1}{8} \\
 \hline
 \text{Ans. } 19537\frac{1}{8} \quad \text{product by } 19\frac{7}{8}
 \end{array}$$

$$37. \quad 147 \times 99\frac{1}{2}$$

$$42. \quad 227 \times 79\frac{7}{8}$$

$$38. \quad 357 \times 29\frac{4}{5}$$

$$43. \quad 670 \times 49\frac{1}{2}$$

$$39. \quad 826 \times 99\frac{3}{4}$$

$$44. \quad 659 \times 89\frac{2}{3}$$

$$40. \quad 980 \times 39\frac{5}{6}$$

$$45. \quad 768 \times 49\frac{3}{4}$$

$$41. \quad 347 \times 99\frac{7}{8}$$

$$46. \quad 538 \times 19\frac{7}{8}$$

277. Miscellaneous Oral Problems.

1. At the rate of $2\frac{1}{3}$ miles per hour, how long will it take a man to walk $5\frac{2}{3}$ miles?
2. At $2\frac{1}{2}\%$ an agent receives \$15 for collecting a bill. How much does he remit to his employer?
3. At what rate per cent will any principal double itself in 15 years?
4. One and one half times 40 is $\frac{5}{12}$ of what number?
5. What per cent of $6\frac{2}{3}$ is 4?
6. What per cent of 4 is $6\frac{2}{3}$?
7. Bought $4\frac{1}{4}$ dozen handkerchiefs at 25 cents each and sold them at $33\frac{1}{3}$ cents each. What did I gain?
8. After spending $\frac{5}{8}$ of my money and giving away $\frac{2}{3}$ of the remainder I had \$60. What did I have at first?
9. A man paid $\frac{3}{10}$ of his money for clothes, \$30 for rent, and had $\frac{1}{2}$ of his money left. How much money had he at first?
10. A dealer marked a pair of shoes 25% above cost. If he takes off 10% of the marked price, what per cent does he gain?
11. What is the cost of 2250 pounds of coal at \$6 per ton of 2000 pounds?
12. The owner of a house pays \$45 for insurance for 3 years on \$3000. What is the rate per cent per year?
13. How long will it take \$25 to earn \$5 at 5%?
14. What must be paid for the use of \$400 for 1 year 1 month 15 days at 6%?
15. $\$832 \div 104\%$ equals what?
16. When the cost is $\frac{3}{4}$ of the selling price, what is the gain per cent?
17. When the selling price is $\frac{3}{4}$ of the cost, what is the loss per cent?

18. The cost of $\frac{3}{8}$ of anything is what part of the cost of one half of it?

19. A sum of money loaned at $3\frac{1}{6}\%$, amounts in 6 years to \$847. How much was loaned?

20. What must I pay for 4% stock to get 5% on the investment? (No brokerage.)

21. If $\frac{2}{3}$ of the value of a field is \$200, what is $\frac{3}{4}$ of the value?

22. What per cent of $\frac{2}{3}$ is $\frac{3}{4}$?

23. What per cent of $\frac{3}{4}$ is $\frac{2}{3}$?

24. A 90-day note for \$600 is made and discounted to-day at 6% . Find the proceeds.

25. How many bolts can be cut from an iron rod $7\frac{1}{2}$ feet long, $\frac{3}{4}$ inch being used for each bolt?

26. How much is received for a ton of coal sold at 30 cents per basket of 80 pounds?

27. Find the value of 1122 sheep at \$3 each.

28. What part of 8 is $\frac{4}{5}$?

29. What is received for an article sold at \$5 less 25% ?

30. A dealer sold an article for \$8.10, thereby losing 10% . At what selling price would he have gained 10% ?

31. A and B have an equal number of cows; A sells 25% of his to B. The number B then owns is what per cent greater than the number A has left?

32. At $\frac{4}{3}$ bushel to a cubic foot, how many cubic feet will contain 80 bushels?

33. If goods costing \$240 are sold at an advance of 125% , what is the selling price?

34. A 63-gallon cask is $\frac{2}{3}$ full. What part of the cask is empty after $10\frac{1}{2}$ gallons are drawn off?

35. Multiply $33.33\frac{1}{3}$ by 18.9.

36. At $\frac{4}{5}$ bushel to a cubic foot, find the capacity in bushels of a bin 7 ft. by $2\frac{1}{2}$ ft. by 2 ft.

37. Find the area of a field 125 rods long by .72 rods wide.

38. In what time will a sum of money double itself at $4\frac{1}{2}\%$ simple interest?

39. A storekeeper sold 12 dozen eggs for \$1.80, gaining 25%. What did they cost per dozen?

40. How many square yards in a piece of ground 44 yards wide and 110 yards long?

278. Miscellaneous Written Problems.

1. Divide 30 and 6 hundredths by the sum of sixteen hundredths and two thousandths.

2. The annual expenses of a school district are:

Teacher's wages	\$ 495
Fuel	51
Janitor's wages	36
Repairs	72
Books	64
Miscellaneous	92

What will be the school tax of Mr. Pennea, whose property is assessed at \$3240, the assessed value of the district being \$40,500?

3. Write in words; 1409.0071.

4. A man buys a piece of property for \$85,010. What monthly rental must he charge to net $7\frac{1}{2}\%$ on this investment and his expenses for repairs, taxes, and insurance, which amount to $4\frac{1}{2}\%$ per year on the cost of the property?

5. What is the exact interest on \$730 from July 1, 1908 to Sept. 23, 1908, at $5\frac{1}{2}\%$?

6. The amount of a certain principal for six years is \$650 and the interest is three tenths of the principal. Find the principal and the rate per cent.

7. A and B set out from the same place and travel in opposite directions, A starting $2\frac{1}{2}$ hours before B. A travels $4\frac{3}{8}$ miles per hour and B travels $3\frac{4}{5}$ miles per hour. How far apart will they be when B has traveled 5 hours and 20 minutes?

8. If a man sells $\frac{2}{3}$ of his farm for what $\frac{3}{4}$ of it cost, what per cent does he gain or lose by the transaction?

9. Find the rate at which \$ 900 in 3 years 8 months will yield \$ 298 interest.

10. Reduce to a whole or a mixed number:

$$\frac{63\frac{1}{2} + (\frac{3}{4} \text{ of } 7\frac{1}{3})}{(\frac{5}{9} \text{ of } 16\frac{1}{5}) - (\frac{5}{6} \text{ of } 3\frac{1}{2})}.$$

11. What will be the cost of 1575 pounds of hay at \$ 5.50 per ton?

12. A man bought a horse and a wagon for \$ 560, and two fifths of the cost of the wagon was equal to two thirds of the cost of the horse. What did he pay for each?

13. A stack of hay will last a cow 8 weeks and a horse 6 weeks. How many days would it last both?

14. A man left $\frac{5}{11}$ of his property to his wife, $\frac{2}{3}$ of the remainder to his son, and the balance, \$ 4000, to his daughter. What was the value of his property?

15. Sold a horse for 40% advance on its cost and with the money bought another. The latter was sold for \$ 224, which was 20% less than its cost. What was the cost of the first horse?

16. Divide \$ 2380 between A and B so that $\frac{2}{3}$ of A's share will be equal to B's.

17. If 6 be subtracted from each term of the fraction $\frac{9}{10}$, by what per cent will the value of the fraction be increased or diminished?

18. A note for \$2000 at 6% dated Jan. 3, 1907, has the following indorsement: July 3, 1907, Received \$800. Find the balance due to-day.

19. A drover buys 15 horses at \$125 each and 25 cows at \$48 each. He loses \$210 on the sale of the horses, but he obtains from the sale of the cows a sum sufficient to yield him a profit of 5% on the whole investment. What was the average price received for the cows?

20. How much must be invested in 5% bonds at $115\frac{1}{2}$, brokerage $\frac{1}{8}\%$, to secure an annual income of \$800?

21. A, B, and C can do together $\frac{1}{4}$ of a piece of work in a day. A alone can do $\frac{1}{10}$ of it in a day, and B alone $\frac{1}{12}$ of it in a day. What part of it can C do alone in a day? How many days would C require to do the whole work?

22. Goods amounting to \$487.50 were bought subject to a certain discount for cash. The sum paid by the purchaser was \$477.75. What was the rate of discount?

23. After selling 75% of a stock of goods at 25% profit, a merchant sells the remainder at a loss of $33\frac{1}{3}\%$. What is the gain or loss per cent on the entire stock?

24. Write 1908 in Roman notation.

25. Express in common fractions and reduce to lowest terms: $.028\frac{4}{7}$, $\frac{2}{5}\%$, 216% .

26. A 3-months note for \$960 is made and discounted to-day at a bank. Find the proceeds.

27. What is the value of my house if my annual premium at $\frac{3}{4}\%$ on $\frac{2}{3}$ of its value is \$40?

28. A farmer has 40 acres in clover which average 10 tons per acre when cut. The hay weighs 48% less when it is placed in the barn. When it is sold 8 months later, it weighs 12% less than it did when placed in the barn. What was the weight of the hay at the time of sale?

29. Divide \$810 among A, B, and C, so that A's share may equal 75% of B's, and B's share may equal 20% of C's.

30. A boy bought a certain number of apples at 2 for a cent and the same number at 3 for a cent, paying 50 cents for all. How many apples did he buy?

31. A boy bought 120 peaches at 3 for a cent and the same number at 2 for a cent. By selling them all at 5 for 2 cents, how much did he lose?

32. The cost of a horse equals how many eighths of 40% of its cost?

33. A creditor receives \$1.50 for every \$4 due him, and thereby loses \$602.10. What was the sum due?

34. How many tons of hay at \$8.50 per ton must a farmer give in exchange for 13,750 feet of hardwood flooring at \$36 per 1000 feet?

35. What will be the difference in annual income between \$10,752 invested in 3% bonds at 95 $\frac{1}{8}$ and brokerage, and the same sum invested in 4% bonds at 111 $\frac{1}{8}$ and brokerage?

36. Which will yield the greater annual income, 3 per cents bought at 96, including brokerage, or 4 per cents bought at 112, including brokerage? What per cent greater?

37. Simplify $\frac{4\frac{2}{3} - (6\frac{2}{3} \times \frac{3}{5})}{2\frac{3}{4} + (8\frac{3}{4} - 13\frac{1}{8})}$.

38. A merchant sells an overcoat for \$22, a suit for \$23, and a hat for \$5. He gains 10% on the overcoat, 15% on the suit, and 25% on the hat. What per cent of the total cost of the articles does he realize on the entire sale?

39. A drover sold 24 horses for \$150 each, gaining $\frac{3}{10}$ of the cost on 12 of them and losing $\frac{1}{5}$ of the cost on the remainder. Find the total gain or loss.

40. My profit on a sale of apples amounting to \$455 was 30%. What per cent would I have lost if the apples had cost \$300 more?

41. Find the price of a 6% stock that will yield as much income as $4\frac{1}{2}\%$ stock bought at par (100). No brokerage.

42. If three eighths of a quantity costs six and three fourths dollars, what will five ninths of the quantity cost?

43. The owner of a house insures it for $\frac{5}{6}$ of its value at the rate of \$75 for three years. What is the rate per year, the house being worth \$12,000?

44. (a) Write in figures: twelve hundred (units) and eight thousand three ten-millionths.

(b) Express in words: 20,000.002.

45. One half the sum of $2\frac{1}{3}$ and $3\frac{1}{2}$ is how many times one third of their difference?

46. An agent sold cotton on commission at 5%, and invested the proceeds in sugar at 2% commission, which was \$57. What was the cost of the sugar? What did he receive for the cotton?

47. (a) Divide .072 by .0004 $\frac{1}{2}$.

(b) Multiply 4 hundredths by 7 hundred-thousandths.

(c) Divide 1 and 92 hundredths by 16 ten-thousandths.

48. A farm mortgaged for 40% of its cost is sold for \$8000, which is $14\frac{2}{3}\%$ more than the cost. What remains from the sale after the mortgage is paid and one half year's interest at 6%?

49. What income will be derived from \$6500 invested in 7% stock at 130? (No brokerage.)

50. A broker's commission for selling bonds was \$62.50. What did the seller receive, the rate being 130, less brokerage of $\frac{1}{8}\%$?

51. At what rate will \$980 amount in 270 days to \$1011.85?

52. What is a merchant's net profit on goods sold to the amount of \$6000, which is $33\frac{1}{3}\%$ above their cost, if he pays a commission of 10% and other expenses of \$800?

53. Divide \$2600 among three persons so that the second shall receive $33\frac{1}{3}\%$ more than the third and the first shall receive 50% more than the second. What per cent does the first get more than the third?

54. After spending $\frac{1}{4}$ of his money, then $\frac{2}{5}$ of the remainder, then $\frac{1}{10}$ of what still remained, a man had \$648. What had he at first?

55. The quotient is 123.4, the divisor 42, the remainder 6. Find the dividend.

56. On a house costing \$6000 there are the following expenses annually: Insurance for $\frac{3}{4}$ of its value at $\frac{1}{2}\%$; taxes \$48; repairs, etc., \$47.50. What rent must the owner receive per month to pay expenses and 6% on the cost of the house?

57. Find the difference between the exact interest and the interest by the common method on \$120,000 at 6% from March 1 to July 25.

58. Simplify $\frac{\frac{8}{9} \times \frac{1}{2}}{1\frac{1}{3} \times \frac{1}{5}} \div \frac{1\frac{1}{2} \times \frac{8}{9}}{\frac{4}{5} \text{ of } 4}$.

59. If wool costing 20¢ per pound loses 20% of its weight in scouring, at what price per pound must the scoured wool be sold to gain 20%?

60. Write an interest-bearing promissory note for \$500, dated July 10, 1907, maturing to-day with interest at 6%. Find the amount due to-day.

61. What amount invested in U.S. 3's at 115, brokerage $\frac{1}{8}\%$, will yield an annual income of \$1200?

62. A dealer imports 1250 pounds of cheese at 13¢ per lb. (duty 8¢ per lb.), 1000 gross of matches at 40¢ per gross (duty at 8¢ per gross), and 300 pounds of preserved fruit at

80¢ per lb. (duty 1¢ per lb. and 35%). Find the total cost, including duty.

63. If \$324 is paid for a piano at 40, 25, and 10% discount, what is the list price?

64. A dealer sold a piano marked \$520 for \$460. If the marked price is 30% above the cost, what per cent did he gain?

65. A and B together have \$500; A and C have together \$600. How much has C more than B? If B and C have together \$700, how much has each? How much has A?

66. A farmer sold a horse, a cow, and a sheep. The price of the horse and the cow was \$200; of the horse and the sheep \$160; of the cow and the sheep \$60. What was the price of each?

67. A man left his property worth \$20,000 to his wife, his daughter, and his son, the daughter's share being 20% more than the son's, and his wife's share being 50% more than the daughter's. What was the share of each?

68. A man has three casks. The second has one third of the capacity of the first, and the third has four ninths of the capacity of the first. If the contents of the other two are poured into the first, it will still hold 8 gallons. What is the capacity of each if the capacity of the first is 8 gallons greater than the combined capacity of the other two?

69. A New York banker shipped \$48,665 in gold to London to settle an account amounting to £10,000. He paid $\frac{1}{8}\%$ freight and $\frac{1}{8}\%$ for insurance. There was a loss of $\frac{1}{16}\%$ by abrasion on \$20,000 in \$20 gold pieces, of $\frac{1}{8}\%$ on \$20,000 in \$10 gold pieces, and of $\frac{1}{4}\%$ on the \$5 gold pieces, which constituted the remainder of the shipment. What was the total cost to the banker, including the sum paid to replace the loss by abrasion?

70. Make out a bill to J. F. Fagan for five articles bought from C. E. Wise & Co. Receipt it.

71. A real estate agent sold a building for \$10,000. Find his commission at 5% on the first \$1000; $2\frac{1}{2}\%$ on \$4000, and $1\frac{1}{4}\%$ on the remainder.

72. Water in freezing expands 10% in bulk. If a cubic foot of water weighs 1000 ounces, what will be the weight of a cubic foot of ice?

73. A real estate agent whose rates are 5% on the first \$1000, $2\frac{1}{2}\%$ on all above \$1000 and not exceeding \$5000, and $1\frac{1}{4}\%$ on all above \$5000, received a commission of \$243.75 for selling a farm. What price was paid for the farm?

74. Write one thousand forty-five and nine millionths. Write seven and nine thousandths.

75. Simplify
$$\frac{63\frac{1}{2} + (\frac{3}{4} \text{ of } 7\frac{1}{3})}{(\frac{5}{9} \text{ of } 16\frac{1}{5}) - (\frac{5}{6} \text{ of } 3\frac{1}{2})}.$$

76. A furniture dealer sold two desks for \$30 and \$28 respectively. On the first he lost $16\frac{2}{3}\%$, and on the other he gained $16\frac{2}{3}\%$. What per cent of loss or of gain was there on the entire transaction?

77. Mr. Mard bought a house for \$4500, paying \$900 in cash and giving three equal notes for the balance payable in 4, 8, and 12 months respectively, with interest at 6%. What was the total sum paid, including interest?

78. An agent received a commission of \$223 on sales amounting for a month to \$1784. What should his commission be the following month if he sold goods to the amount of \$1496?

79. A dealer bought 6 dozen oranges at 15 cents per dozen. He sold $\frac{1}{2}$ of them at 2 for 5 cents, $\frac{1}{3}$ of them at 1 cent each, and the remainder at 3 for 5 cents. What per cent did he gain?

80. Compile a compound-interest table showing the compound interest on \$1 for 1 year, 2 years, 3 years, 4 years, at 1%, 2%, 3%, 4%, and 5%, to four places of decimals.

81. A miller purchased 1200 bushels of wheat through an agent, at 95¢ per bushel. The total cost to the miller, including freight charges of \$60 and the agent's commission, was \$1248. How much per bushel was the commission?

82. Find the difference between the accurate interest and the regular interest on \$800 from April 1 to Nov. 6 at 5%.

83. The assessed valuation of a town is \$875,000. The whole amount of taxes to be raised is \$13,125. What is A's tax, if his property is assessed at \$1680?

84. Write as common fractions :

(a) .025. (b) $\frac{3}{4}$ of one per cent. (c) $\frac{1}{9}\%$. (d) $6\frac{1}{3}\%$.

Change the following to expressions having the per cent sign :

(e) .073. (f) $\frac{3}{8}$. (g) $.00\frac{1}{3}$. (h) $\frac{2}{3}$.

85. A shipper sent a commission merchant 2100 bushels of wheat to sell, and instructed him to invest the net proceeds in salt at \$2 per barrel. The commission merchant deducted 2% commission for selling the wheat and sent the shipper 980 barrels of salt, for the purchase of which he charged a commission of 5%. What price per bushel was obtained for the wheat?

86. By selling at 20% less than his asking price, a merchant makes a profit of 20%. What per cent profit would he have made if he had obtained his asking price?

87. A man bought a watch and a chain for \$70. One half of the cost of the watch equals two thirds of the cost of the chain. Find the cost of each.

88. What per cent is gained by buying at the rate of 3 yards for \$2 and selling at the rate of 2 yards for \$3?

89. A buys stock at 78 and sells it at 84. B buys bonds at 70 and sells at 75. A makes \$1000 more than B. How much did A invest? (No brokerage.)

90. A bin 6 feet long 4 feet wide holds 72 bushels. How deep is the bin, assuming that 1 cubic foot equals $\frac{4}{3}$ bushel?

91. A dealer sold $\frac{3}{8}$ of a quantity of cloth at a profit of 20% and the remainder at cost. What per cent did he gain?

92. A note for \$7200 dated Sept. 1, 1906, with interest at 6% bears the following indorsements:

Jan. 2, 1907, \$1250; July 25, 1907, \$45; Dec. 26, 1907, \$975; May 1, 1908, \$600. Find the amount due July 3, 1908.

93. Reduce $.1944\frac{4}{9}$ to a simple fraction in its lowest terms.

94. If 18 pipes each delivering 6 gallons per minute fill a cistern in 2 hours and 16 minutes, how many pipes, each delivering 20 gallons per minute, will fill a cistern $7\frac{1}{2}$ times as large in 3 hours 24 minutes?

95. A man sells 45 shares of stock at $118\frac{1}{8}$, and invests in $3\frac{1}{2}\%$ bonds at $89\frac{7}{8}$, brokerage $\frac{1}{8}\%$ in each case. If the stock paid 5%, by how much is his yearly income changed? How does the certainty of his income from the bonds compare with that from his investment in stock?

96. Three fourths of A's farm is equal in value to two thirds of the value of B's. Together, the farms are worth \$13,600. What is the value of each?

97. A invests \$1200 and makes \$500 in four months. B invests \$750 and makes \$250 in five months. Which makes the greater per cent a month on his money, and how much?

98. Find the interest on \$2500 from Sept. 13, 1905, to May 4, 1908, at 5 per cent.

99. By selling an article marked \$60 at a discount of 20%, a profit of 20% is made. What did the article cost the seller?

100. What is the cost of 19,250 pounds of coal at \$6 per ton of 2000 pounds?

101. What per cent must be added to the cost to leave a profit of 20% after a discount of 25%?

102.
$$\frac{(3.75 + 19.867) \times (82.5 - 28.425)}{10.815}.$$

103. Find the cost of a cable transfer of £ 426 16s. at \$4.8735 per £.

CHAPTER V.

DENOMINATE NUMBERS; MEASUREMENTS.

REVIEW OF DENOMINATE NUMBERS.

279. Preliminary Exercises.

English Money.

4 farthings = 1 penny (*d.*)

12 pence = 1 shilling (*s.*)

20 shillings = 1 pound (*£*)

1. How many pence in 4 shillings?
2. Change 4 shillings 4 pence to pence.
3. Reduce £ 21 to shillings.
4. How many shillings in £ 2 10*s.*?
5. Change 50 shillings to pence.
6. How many pence in 50*s.* 6*d.*?
7. How many shillings in £ $\frac{1}{4}$?
8. Reduce £ $\frac{1}{8}$ to shillings and pence.
9. How many shillings in .25 of £?
10. How many shillings and pence in .125 of £?
11. Change £ $\frac{3}{8}$ to shillings and pence.

REDUCTION DESCENDING.

280. Written Exercises.

1. Reduce £ 24 17*s.* 6*d.* to pence.

$$\begin{array}{rcl}
 £\ 24 & =\ 20*s.* \times 24 & =\ 480*s.* \\
 & & +\ 17*s.* \\
 £\ 24\ 17*s.* & & =\ \underline{497*s.*} \\
 £\ 24\ 17*s.* & =\ 12*d.* \times 497 & =\ 5964*d.* \\
 & & +\ 6*d.* \\
 £\ 24\ 17*s.* 6*d.* & & =\ \underline{5970*d.*} \quad Ans.
 \end{array}$$

Since there are 20 shillings in a pound, in £24 there are 24 times 20 shillings, or 480 shillings. In £24 17s. there are 480 shillings + 17 shillings, or 497 shillings.

Since there are 12 pence in a shilling, in 497 shillings there are 497 times 12 pence, or 5964 pence. In 497s. 6d. (£24 17s. 6d.) there are 5964 pence + 6 pence, or 5970 pence. *Ans.* 5970d.

<u>20s. 12d.</u> £24 17s. 6d. <u>497s. 6d.</u> <i>Ans.</i> 5970d.	The accompanying arrangement of the work is suggested. Above 17s. write the number of shillings (20s.) in £1; above 6d. write the number of pence (12d.) in a shilling.
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Multiply 20s. by 24, and add in 17s. at the same time, which gives 497s. for the first step. Multiply 12d. by 497 and add in 6d. for the final result.

In practice, however, 20 and 12 are employed as the multipliers.

2. Reduce 47 gal. 1 pt. to pints.

<u>4 qt. 2 pt.</u> 47 gal. 0 qt. 1 pt. <u>188 qt. 1 pt.</u> <i>Ans.</i> 377 pt.	Write 0 qt. in the proper place. Above the column of quarts write 4 qt., the number of quarts in a gallon, and above the column of pints write 2 pt., the number of pints in a quart.
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Reduce:

3. £47 15s. 10d. to pence.
4. £33 8d. to pence.
5. £59 16s. to pence.
6. 27 gal. 1 pt. to pints.
7. 14 gal. 3 qt. to pints.
8. 13 bu. 2 pk. 3 qt. to quarts.
9. 27 yd. 1 ft. 9 in. to inches.
10. 3 da. 15 hr. 40 min. to minutes.

281. Troy Weight.

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

Troy weight is used in weighing gold and silver.

1 lb. troy	= 5760 grains.
1 lb. avoirdupois	= 7000 grains.

Reduce:

1. 15 pwt. 17 gr. to grains.
2. 12 ounces troy to grains.
3. $\frac{3}{4}$ lb. avoirdupois to grains.
4. 1 oz. avoirdupois to grains.
5. 3 lb. 7 oz. 12 pwt. 16 gr. to grains.
6. 1 lb. troy to grains.
7. 1 lb. troy to fraction of avoirdupois pound.
8. 1 oz. avoirdupois to fraction of troy ounce.
9. 1 lb. avoirdupois to lb., oz., etc., troy.
10. 2 lb. 4 pwt. to grains.

REDUCTION ASCENDING.

282. Preliminary Exercises.

Reduce:

1. 48 pence to shillings.
2. 52 pence to shillings and pence.
3. 120 shillings to £.
4. 135 shillings to £, etc.
5. 240 pence to £.
6. 250 pence to £, etc.
7. 2s. 6d. to the fraction of a £.
8. 7s. 6d. to the decimal of a £.
9. 480 grains to ounces.
10. 490 grains to ounces and grains.
11. 60 grains to the decimal of an ounce.
12. .125 lb. to pennyweights and grains.
13. $\frac{3}{8}$ lb. to pennyweights and grains.
14. 96 quarts to bushels.
15. 125 pounds to the fraction of a ton.

283. Written Exercises.

1. Reduce 5970*d.* to higher denominations.

20 <i>s.</i>	12 <i>d.</i>
<hr/>	
	5970 <i>d.</i>
497 <i>s.</i>	6 <i>d.</i>
£ 24	17 <i>s.</i> 6 <i>d.</i>

Above 5970*d.* write 12*d.*, the equivalent of 1*s.* in pence. Dividing 5970*d.* by 12*d.*, we obtain a quotient of 497, the number of shillings, and a remainder of 6*d.*

The number of pounds sterling in 497*s.* is obtained by dividing by 20*s.*, which gives 24 and a remainder of 17*s.*

2. Reduce 377 pints to higher denominations.

4 qt.	2 pt.
<hr/>	
	377 pt.
188 qt.	1 pt.
47 gal.	1 pt. Ans.

Reduce to higher denominations:

- | | |
|-----------------------|---|
| 3. 5400 grains. | 11. 472 qt. (dry). |
| 4. 3600 pwt. | 12. 288 gr. to fraction of lb. |
| 5. 4000 grains. | 13. 576 gr. to decimal of lb. |
| 6. 2740 pence. | 14. 200 <i>d.</i> to fraction of £. |
| 7. 1500 pence. | 15. 17 <i>s.</i> 6 <i>d.</i> to decimal of £. |
| 8. 3750 minutes. | 16. 1 hr. 30 min. to fraction of da. |
| 9. 3750 seconds. | 17. 13 hr. 30 min. to decimal of da. |
| 10. 356 pt. (liquid). | 18. 3 qt. to decimal of bu. |

OPERATIONS IN DENOMINATE NUMBERS.

284. Sight Exercises.

- | | | |
|-------------------------------------|---|----------------------------|
| 1. 24 lb. 8 oz.
+ 24 lb. 8 oz. | 2. 49 lb.
- 24 lb. 8 oz. | 3. 24 lb. 8 oz.
× 2 |
| 4. 2)49 lb. | 5. 24 lb. 8 oz.) 49 lb. | 6. $\frac{1}{2}$ of 17 lb. |
| 7. 3 qt. 1 pt.
+ 5 qt. 1 pt. | 8. 8 gal.
- 3 gal. 3 qt. | 9. 10 gal. 3 qt.
× 3 |
| 10. 4)£ 20 16 <i>s.</i> 8 <i>d.</i> | 11. $\frac{1}{3}$ of 9 oz. 15 pwt. 18 gr. | |

285. Written Exercises.

1. Add £ 24 16s. 8d., £ 37 9d., 17s. 6½d., £ 42 16s.

£ 24	16s.	8 d.	8d. + 9d. + 6½d. = 23½d. = 1s. 11½d. Write
37		9	11½d. and carry 1s.
	17	6½	1s. + 16s. + 17s. + 16s. = 50s. = £ 2 10s. Write
42	16		10s. and carry £ 2.
£ 105	10s.	11½d.	£ 2 + £ 42 + £ 37 + £ 24 = £ 105.

Add:

2. £ 2 10s. 6d., 17s. 8d., £ 24 13s.

3. 14 lb. 6 oz., 27 lb. 10 oz., 17 oz., 19 lb. 12 oz. (avoirdupois).

4. 3 bu. 2 pk. 7 qt., 5 bu. 3 pk., 2 pk. 6 qt.

5. 13 gal. 3 qt. 1 pt., 17 gal. 2 qt., 3 qt. 1 pt., 25 gal.

6. 14 lb. 6 oz. 17 pwt., 29 lb. 8 oz., 10 oz. 8 pwt., 4 lb. 15 pwt.

7. 7 yd. 1 ft. 9 in., 18 yd. 2 ft. 10 in., 25 yd. 1 ft. 6 in.

8. £ 33 17s. 6d., £ 24 15s. 8d., £ 19 15s. 11d.

9. 3 oz. 13 pwt., 8 oz. 15 pwt. 18 gr., 19 pwt. 10 gr., 15 gr.

10. 24 bu. 3 pk., 2 bu. 2 pk. 2 qt., 3 pk. 7 qt., 4 qt.

11. From 32 bu. 1 pk. 1 qt. take 14 bu. 3 pk. 5 qt.

32 bu. 1 pk. 1 qt.	Since 5 qt. is greater than 1 qt., we
14 bu. 3 pk. 5 qt.	borrow 1 pk., or 8 qt., making 9 qt.
17 bu. 1 pk. 4 qt.	5 qt. from 9 qt. leaves 4 qt., etc.

12. £ 22 10s. 6d. — £ 4 17s. 8d.

13. 23 bu. 1 pk. 1 qt. — 15 bu. 3 pk. 6 qt.

14. 145 gal. 1 qt. — 24 gal. 3 qt. 1 pt.

15. 23 lb. 9 oz. 5 pwt. — 5 lb. 10 oz. 18 pwt.

16. 115 oz. 10 pwt. 12 gr. — 24 oz. 15 pwt. 20 gr.

17. 240 yd. 1 ft. 6 in. — 115 yd. 2 ft. 9 in.

18. £ 100 6d. — £ 20 15s. 9d.

19. 63 gal. — 42 gal. 2 qt. 1 pt.

20. 123 yd. 7 in. — 60 yd. 2 ft. 9 in.

21. Multiply 4 wk. 18 hr. 30 min. by 5.

4 da. 18 hr. 30 min.	30 min. $\times 5 = 150$ min. = 2 hr. 30 min.
<u>5</u>	Write 30 min. and carry 2 hr.
23 da. 20 hr. 30 min.	18 hr. $\times 5 = 90$ hr. Add 2 hr., making 92 hr., which equals 3 da. 20 hr.

Write 20 hr. and carry 3 da.

4 da. $\times 5 = 20$ da. Add 3 da., making 23 da.

22. 3 wk. 4 da. 12 hr. $\times 6$.

23. 14 bu. 2 pk. 7 qt. $\times 8$.

24. £15 17s. 6d. $\times 7$.

25. 3 oz. 15 pwt. 12 gr. $\times 9$.

26. 17 gal. 1 qt. 1 pt. $\times 4$.

27. 153 yd. 2 ft. 9 in. $\times 3$.

28. £43 9s. 4d. $\times 12$.

29. 7 oz. 12 pwt. 10 gr. $\times 5$.

30. 24 bu. 2 pk. 3 qt. $\times 11$.

31. Divide 23 da. 20 hr. 30 min. by 5.

5)23 da. 20 hr. 30 min.	23 da. $\div 5 = 4$ da. and 3 da. remainder. Write 4 da.
4 da. 18 hr. 30 min.	3 da. = 72 hr. 72 hr. + 20 hr. = 92

hr. 92 hr. $\div 5 = 18$ hr. and 2 hr. remainder. Write 18 hr.

2 hr. = 120 min. 120 min. + 30 min. = 150 min. 150 min. $\div 5 = 30$ min. Write 30 min.

32. 247 days $\div 8$.

33. 139 troy ounces $\div 6$.

34. $\frac{1}{3}$ of 23 da. 20 hr. 30 min.

35. $\frac{2}{3}$ of 11 da. 22 hr. 15 min.

36. $\frac{1}{4}$ of 175 bu. 3 pk. 4 qt.

37. $\frac{3}{4}$ of £27 17s. 4d.

38. .75 of £55 14s. 8d.

39. .125 of 43 bushels.

40. .375 of £250.

41. Divide 23 da. 20 hr. 30 min. by 4 da. 18 hr. 30 min.

When the divisor and the dividend are concrete numbers, they must have the same denomination. Reducing both terms to minutes, we have,

$$23 \text{ da. } 20 \text{ hr. } 30 \text{ min.} = 34350 \text{ min.}$$

$$4 \text{ da. } 18 \text{ hr. } 30 \text{ min.} = 6870 \text{ min.}$$

$$34350 \text{ min.} \div 6870 \text{ min.} = 5.$$

That is, 34350 minutes contains 6870 minutes 5 times.

42. Divide £106 by £6 12s. 6d.

$$£106 = 25440d.$$

$$£6 \text{ } 12s. \text{ } 6d. = 1590d.$$

$$25440d \div 1590d = 16. \text{ } Ans.$$

The divisor may be reduced to £6 $\frac{5}{8}$, in which case the problem becomes,

$$£106 \div £6\frac{5}{8} = 106 \div 6\frac{5}{8} = 106 \div \frac{53}{8} = 106 \times \frac{8}{53}. \text{ } Cancel.$$

NOTE. When the divisor and the dividend are concrete, the quotient is abstract.

43. £30 ÷ £3 15s.

44. £30 ÷ 2s. 6d.

45. 1 da. ÷ 1 hr. 36 min.

46. 2 oz. ÷ 1 pwt. 16 gr.

47. 23 gal. 1 qt. 1 pt. ÷ 1 gal. 1 qt. 1 pt.

48. 12 yd. 2 ft. 6 in. ÷ 1 yd. 2 ft. 6 in.

49. 46 bu. 4 qt. ÷ 3 bu. 3 pk. 3 qt.

50. 22 hr. 32 min. 30 sec. ÷ 2 hr. 15 min. 15 sec.

286. Oral Problems.

1. What is the cost in English money of 8 yards of silk at 2s. 6d. per yard?

2. Eight ounces avoirdupois is what fraction of 4 pounds?

3. How many flag-stones 8 feet long will be required for a walk 120 yards long?

4. Find the time in hours and minutes from 8.30 A.M. to 1.15 P.M.

5. A ten-dollar gold piece weighs $10\frac{3}{4}$ pennyweights. How many grains does it weigh?

6. How many strips of carpet $\frac{3}{4}$ yard wide will be required to cover a floor 18 feet wide?

7. Mr. Tully sells 50 quarts of milk a day. How many gallons does he sell in 30 days?

8. At 60 lb. per bushel, how many bushels and pecks will there be in 375 pounds of wheat?

9. A road is 4 rods wide. What is its width in feet?

10. How many inches in 25 yards?

287. Written Problems.

1. How much is the profit on 126 bushels of potatoes bought at 80 cents a bushel and sold at 15 cents a half peck?

2. Find the average height of 20 boys, 2 of whom measure 5 ft. 5 in. each; 4, 5 ft. 6 in. each; 6, 5 ft. 7 in. each; and 8, 5 ft. 8 in. each.

3. A horse is fed 6 quarts of oats per day. How many bushels, pecks, and quarts will he eat in July?

4. A field is 80 rods long and 20 rods wide. How many feet of wire will be required to inclose it with a fence four strands high?

5. A seed dealer made up 500 half-pint packages of beans. How many bushels, pecks, etc., were required?

6. How many $\frac{1}{2}$ -pt. bottles will be needed to hold 63 gallons of catsup?

7. At \$16 per ton, find the cost of 14 bales of hay averaging 175 pounds each.

8. At \$40 per month, what is the total rent of a house for 5 years 7 months 15 days?

9. What do I pay for the use of a certain sum of money for 3 years 8 months 6 days at the rate of \$60 per year?

10. A dealer buys 500 tons of coal at \$5 per ton of 2240 pounds. He sells it at \$6 per ton of 2000 pounds. What is his profit after deducting \$375 for freight, cartage, etc.?

STANDARD TIME.

288. Problems in longitude and time have but little practical value except for navigators, owing to the almost universal employment of *standard time*.

289. Although no two places not on the same meridian can have the same solar time, the railroads of the United States and Canada agreed in 1883 that time belts should be established approximating 15° in width, and that within these belts the time should be that of 60° , 75° , 90° , 105° , and 120° west longitude, respectively; that is, 4, 5, 6, 7, and 8 hours, respectively, earlier than the time of Greenwich. These time belts, beginning on the east, are known as the Atlantic, Eastern, Central, Mountain, and Pacific. The boundaries of the belts are, as nearly as may be, $7\frac{1}{2}^\circ$ on each side of the meridian, though each railroad changes its time at some important point on its own line. Many of the roads passing through Buffalo change from eastern to central time at that point; the Grand Trunk, however, makes its change at Sarnia, Canada.

290. Greenwich time is used throughout England, Holland, Belgium, and Spain.

Germany employs Central European time, which is one hour later than that of Greenwich. This is also the time of Austria, Hungary, Italy, Switzerland, Norway, and Sweden.

France, Algiers, and Tunis hold to the time of Paris, 9 minutes faster than that of Greenwich.

European Russia, Turkey in Europe, Roumania, and

Bulgaria use the time of St. Petersburg, which is 2 hours 1 minute 13 seconds faster than Greenwich time.

Japan time is 9 hours later than Greenwich. Australia has several time belts.

NOTE. It is suggested that but little attention be given to problems involving solar time.

LONGITUDE AND TIME.

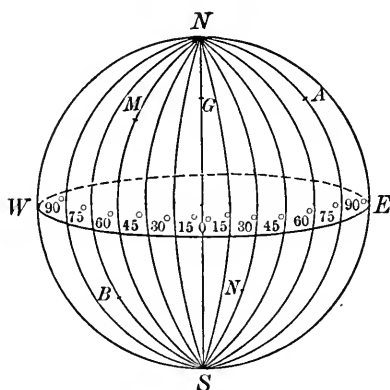
291. Circular Measure.

60 seconds (") = 1 minute (')

60 minutes = 1 degree (°)

360 degrees = 1 circle.

292. By the longitude of a place is meant the distance in degrees measured on the equator between the meridian of the place and the prime meridian.



The meridian of a place is the half of the great circle passing through the place and terminating at the poles. The prime meridian is generally taken as the meridian passing through Greenwich, England. Its longitude is 0°.

The longitude of *A* is 75° east; that of *B*, 60° west; of *M*, 45° west; of *N*, 30° east.

293. Oral Exercises.

Find the difference in longitude between :

- | | | |
|--------------------------|--------------------------|---------------------------|
| 1. <i>A</i> and <i>G</i> | 5. <i>A</i> and <i>B</i> | 9. <i>B</i> and <i>N</i> |
| 2. <i>B</i> and <i>G</i> | 6. <i>A</i> and <i>M</i> | 10. <i>M</i> and <i>N</i> |
| 3. <i>M</i> and <i>G</i> | 7. <i>A</i> and <i>N</i> | |
| 4. <i>N</i> and <i>G</i> | 8. <i>B</i> and <i>M</i> | |

294. Since the earth revolves from west to east, making a complete revolution, 360°, in 24 hours, the sun appears to

travel from east to west at the rate of 15° in 1 hour. A being 75° east of G , it will be noon at A 5 hours earlier than at G .

295. Find the difference in sun time between places whose longitudes are, respectively :

- | | |
|------------------------------------|------------------------------------|
| 1. 75° E. and 0° | 6. 45° W. and 0° |
| 2. 45° W. and 30° E. | 7. 60° W. and 30° E. |
| 3. 75° E. and 30° E. | 8. 75° E. and 60° W. |
| 4. 60° W. and 0° | 9. 60° W. and 45° W. |
| 5. 75° E. and 45° E. | 10. 30° E. and 0° |

NOTE. The more easterly place has the later time.

296. Oral Exercises.

When it is noon at G , find the time at :

- | | |
|------------------------|------------------------|
| 1. A , 75° E. | 3. M , 45° W. |
| 2. B , 60° W. | 4. N , 30° E. |

When it is noon at A , 75° E., find the time at :

- | | |
|------------------------|------------------------|
| 5. G , 0° | 7. M , 45° W. |
| 6. B , 60° W. | 8. N , 30° E. |

When it is noon at B , 60° W., find the time at :

- | | |
|-------------------------|-------------------------|
| 9. G , 0° | 11. M , 45° W. |
| 10. A , 75° E. | 12. N , 30° E. |

Find the longitude or the time :

LONGITUDE OF X .	LONGITUDE OF Y .	TIME AT X .	TIME AT Y .
13. ?	45° E.	9 A.M.	1 P.M.
14. 75° W.	?	2 P.M.	12 M.
15. 30° E.	45° W.	?	3 P.M.
16. 60° E.	45° E.	7 A.M.	?
17. 75° E.	?	4 P.M.	NOON
18. ?	60° W.	5 A.M.	1 A.M.

	LONGITUDE OF X.	LONGITUDE OF Y.	TIME AT X.	TIME AT Y.
19.	30° E.	45° E.	11 A.M.	?
20.	75° E.	30° W.	?	7 P.M.

SOLAR TIME.

297. To determine the longitude of a vessel, the captain notes the time shown by his chronometer, when his observation of the sun shows the ship's time to be exactly noon. The chronometer gives the time at Greenwich. If the chronometer marks exactly 1 o'clock, the vessel is 15° west of the meridian of Greenwich; if it marks 10 o'clock, the vessel is 30° east of the meridian of Greenwich.

298. Written Exercises.

1. Find the longitude of a vessel when the chronometer indicates 12 minutes 30 seconds past 2 at the moment the captain's observation of the sun determines that it is exactly noon.

A difference of 1 hour in time makes a difference of 15° of longitude; a difference of 1 minute of time makes a difference of 15' of longitude; a difference of 1 second of time makes a difference of 15'' of longitude.

$$\begin{aligned}
 15^\circ \times 2 \text{ (the number of hours)} &= 30^\circ \\
 15' \times 12 \text{ (the number of minutes)} &= 3^\circ \\
 15'' \times 30 \text{ (the number of seconds)} &= 7' 30'' \\
 \text{Difference in longitude} &= 33^\circ 7' 30''
 \end{aligned}$$

In practice, the difference in time is written as degrees, minutes, and seconds and multiplied by 15.

$$\begin{array}{r}
 2^\circ 12' 30'' \\
 \times 15 \\
 \hline
 \text{Ans. } 33^\circ 7' 30'' \text{ west.}
 \end{array}$$

As the vessel's time is earlier than that of Greenwich, the vessel is west of Greenwich.

To find the difference in longitude, write the time difference as degrees, minutes, and seconds, and multiply by 15.

2. Find the difference in longitude between two places whose difference in time is 4 hr. 15 min. 25 sec.

3. The difference in time between two places is 3 hr. 20 min. Find the difference in longitude.

4. When it is 12 noon at Greenwich, it is 7:30 A.M. at *B*. What is the longitude of *B*?

5. It is noon at *X* when it is 7:30 A.M. at Greenwich. Find the longitude of *X*.

6. Find the difference in time between two places which differ in longitude $33^{\circ} 7' 30''$.

Since the difference in longitude is obtained by multiplying the time difference by 15, the latter is obtained by dividing the longitude difference by 15, the number of degrees of longitude being changed to hours, etc.

$$\begin{array}{r} 15 \overline{) 33 \text{ hr. } 7 \text{ min. } 30 \text{ sec.}} \\ \underline{2 \text{ hr. } 12 \text{ min. } 30 \text{ sec.}} \\ \text{Ans.} \end{array}$$

To find the difference in time, write the longitude difference as hours, minutes, and seconds, and divide by 15.

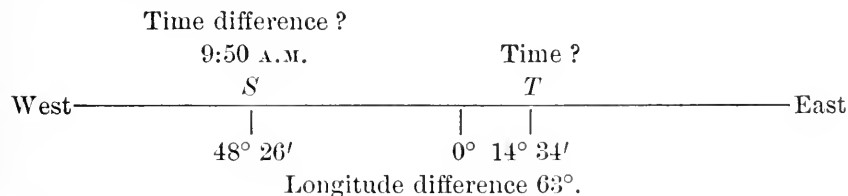
7. The difference in longitude between two places is $48^{\circ} 40' 15''$. Find the difference in time.

8. When it is noon at Greenwich, what is the time at a place in longitude $74^{\circ} 36' 30''$ east?

9. Find the time of a place in longitude $48^{\circ} 27' 45''$ west when it is 12 M. at Greenwich.

10. When it is 9:50 A.M. at *S* in longitude $48^{\circ} 26'$ west, what is the time at *T* in longitude $14^{\circ} 34'$ east?

The employment of a diagram will be of assistance in the solution of problems of this kind.



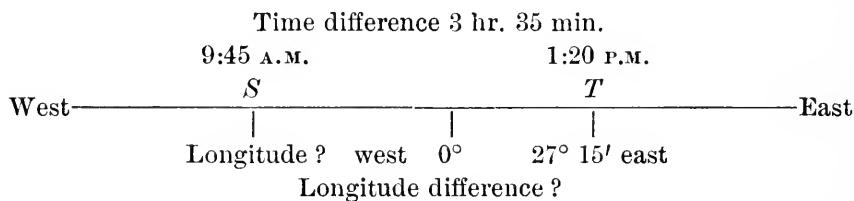
On a line representing the equator locate the prime meridian 0° and the meridian of each of the places, writing under each its longitude and above *S* its time.

To ascertain the time at T , we must add to the time at S the time difference between the places. This is obtained from the longitude difference, which is found to be $63^\circ, 48' 26'' + 14^\circ 34'$.

The time difference is $63 \text{ hr.} \div 15$, or 4 hr. 12 min. Adding 9 hr. 50 min. and 4 hr. 12 min., we have 14 hr. 2 min. The time at T is, therefore, 2 min. past 2 P.M.

NOTE. Difference in longitude is found by adding the two longitudes when one is east and the other is west. The time difference is added to the time of the western place to obtain the time of the more easterly place.

11. When it is 1:20 P.M. at a place in longitude $27^\circ 15'$ east, it is 9:45 A.M. at another place. Find the longitude of the latter.



Since the time of S is earlier than that of T , the former is placed west of the latter, the time difference being 3 hr. 35 min. The longitude difference is $(3^\circ 35' \times 15) 53^\circ 45'$; that is, S is $53^\circ 45'$ west of T . As the latter is $27^\circ 15'$ east of the prime meridian, S must be $26^\circ 30'$ beyond, or $26^\circ 30'$ west.

Find the longitude or the time :

	LONGITUDE OF Q .	LONGITUDE OF R .	TIME AT Q .	TIME AT R .
12.	45° east	45° west	9:30 A.M.	?
13.	12° west	154° west	?	1:20 A.M.
14.	$27^\circ 30'$ east	?	4:15 P.M.	11:30 A.M.
15.	?	$78^\circ 15'$ east	2:30 P.M.	8 P.M.
16.	30° east	?	10 A.M.	2 P.M.
17.	$47^\circ 20'$ west	$15^\circ 40'$ east	?	3:30 P.M.
18.	90° west	50° west	1:30 P.M.	?
19.	?	$87^\circ 30'$ east	12 NOON	7:30 A.M.
20.	$45^\circ 30'$ west	?	9 A.M.	2:40 P.M.

MEASUREMENTS.

AREA OF RECTANGLES.

299. Square Measure.

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)

300. *The number of square inches (feet, yards, etc.) in a rectangle equals the product of the number of inches (feet, yards, etc.) in its length by the number in its width.*

301. Oral Exercises.

Find the area of the following rectangles in square inches :

NOTE. Change each dimension to inches.

- | | |
|---------------------|---------------------------------|
| 1. 30 in. by 20 in. | 6. 10 ft. by 120 in. |
| 2. 42 in. by 20 in. | 7. 1 yd. by 20 in. |
| 3. 48 in. by 25 in. | 8. $1\frac{1}{2}$ ft. by 10 in. |
| 4. 1 ft. by 12 in. | 9. 2 ft. by 30 in. |
| 5. 1 ft. by 1 ft. | 10. 2 ft. 1 in. by 1 yd. |

Find the area in square feet:

NOTE. Change each dimension to feet.

- | | |
|---------------------------|----------------------|
| 11. 3 ft. by 3 ft. | 16. 42 in. by 24 in. |
| 12. 36 in. by 36 in. | 17. 30 ft. by 20 ft. |
| 13. 1 yd. by 1 yd. | 18. 84 ft. by 50 ft. |
| 14. 3 ft. 6 in. by 2 ft. | 19. 12 yd. by 25 ft. |
| 15. 3 ft. 6 in. by 24 in. | 20. 10 yd. by 10 yd. |

Find the area in square yards:

NOTE. Change each dimension to yards.

- | | |
|---------------------------------|----------------------|
| 21. 3 ft. by 3 ft. | 24. 12 ft. by 18 ft. |
| 22. 36 in. by 36 in. | 25. 18 in. by 72 in. |
| 23. $1\frac{1}{2}$ ft. by 6 ft. | 26. 4 yd. by 24 ft. |

27. 18 ft. by 15 ft.

29. 12 yd. by 15 ft.

28. 6 yd. by 7 yd.

30. 30 ft. by 30 ft.

Find the area in square rods:

NOTE. Change each dimension to rods.

31. 20 rd. by 15 rd.

36. 240 rd. by 20 rd.

32. 55 yd. by 10 rd.

37. 160 rd. by $87\frac{1}{2}$ rd.

33. 165 ft. by 55 yd.

38. 125 rd. by 50 rd.

34. 40 rd. by 40 rd.

39. 90 rd. by 30 rd.

35. 168 rd. by 50 rd.

40. 120 rd. by 120 rd.

Find the area in acres:

NOTE. Divide area in square rods by 160.

41. 40 rd. by 4 rd.

46. 80 rd. by 40 rd.

42. 40 rd. by 20 rd.

47. 160 rd. by 110 rd.

43. 40 rd. by 40 rd.

48. 80 rd. by 104 rd.

44. 20 rd. by 20 rd.

49. 124 rd. by 40 rd.

45. 80 rd. by 80 rd.

50. 80 rd. by 36 rd.

302. Oral Problems.

1. How many square inches in the top of a paving brick 8 in. by 4 in. by $2\frac{1}{2}$ in.? In each side? In each end?

2. How many square feet of sidewalk will 144 bricks cover, if each covers a surface of 8 inches by 4 inches?

3. Find the number of square feet of pavement that will be covered by 144 bricks laid on the side 8 inches by $2\frac{1}{2}$ inches.

4. How many shingles will be needed to cover 100 square feet of roof, if each shingle covers a surface of 4 inches by 4 inches?

5. How many bricks will be required to make 100 square yards of pavement, if each brick covers a surface of 8 inches by $2\frac{1}{2}$ inches?

6. The sides of a house are covered with shingles, each of which covers a surface 4 inches by 6 inches. How many shingles are required to the square foot?

7. How many boards 12 feet long and 1 foot wide will be required for a walk 120 feet long, 6 feet wide?

8. How many boards 12 feet long and 8 inches wide will be required for a walk 120 feet long, 6 feet wide?

9. At \$80 per acre, find the cost of a square field, each side of which measures 40 rods.

10. Find the number of square yards in the ceiling of a room 18 feet long, 15 feet wide.

11. How many square yards in each side wall of a room 18 feet long, 9 feet high?

12. Find the area in square yards of each end wall of a room 15 feet wide, 9 feet high.

13. How many square yards of carpet are there in a strip 36 yards long, 27 inches wide?

303. Written Problems.

1. How many acres in a rectangular piece of ground 45 rods long and 24 rods wide?

$$\text{Area in acres} = \frac{45 \times 24}{160}.$$

2. How many square yards can be laid with 900 tiles, each 6 inches square?

3. A room is 18 feet long, 15 feet wide, and 9 feet high. How many square yards in each side wall? In each end wall? In the ceiling? In the four walls and the ceiling?

4. How many square yards of carpet will be required for the floor of a room 18 feet long, 15 feet wide? If the carpet is 1 yard wide, how many running yards are needed? How many yards are needed, if the carpet is $\frac{3}{4}$ yd. wide?

5. How many paving bricks will be required for a

sidewalk 300 feet long and 12 feet wide, the bricks being laid on the broadest face?

6. A street 300 feet long and 36 feet wide is paved with bricks laid on the side. How many are needed?

7. At 15 laths to the square yard, how many will be used for the walls and the ceiling of a room 18 feet long, 15 feet wide, 9 feet high, allowing for two windows, each 6 feet by $4\frac{1}{2}$ feet, and a door 8 feet by $4\frac{1}{2}$ feet?

8. How many boards 16 feet long and 8 inches wide will be needed for the floor of a dock 120 feet long and 24 feet wide?

9. The owner of an inclosed lot 100 feet square has sodded it, excepting a 6-foot walk adjoining the fence. How many square feet of sods will be required? How many square feet of paving will be needed for the walk? How many square feet of paving would be required for a sidewalk adjoining the fence on the outside?

VOLUME OF RECTANGULAR SOLIDS.

304. Cubic Measure.

1728 cubic inches	= 1 cubic foot (cu. ft.)
27 cu. ft.	= 1 cubic yard (cu. yd.)
128 cu. ft.	= 1 cord of wood
231 cu. in.	= 1 gallon
2150.4 cu. in.	= 1 bushel
1 cu. ft. of water	weighs 1000 oz.

305. Oral Problems.

1. At $7\frac{1}{2}$ gallons to the cubic foot, what is the capacity of a cubical tank measuring 2 feet on a side?

2. At $\frac{4}{5}$ bushel to the cubic foot, find the capacity of a cubical bin, each side of which measures 10 feet.

3. How many cubic inches in a brick measuring 8 inches by 4 inches by $2\frac{1}{2}$ inches?

4. What fraction of a cubic foot is the volume of a brick measuring 8 inches by 4 inches by $2\frac{1}{2}$ inches?

5. How many cubic feet of air are there in a room 20 feet long, 15 feet wide, 9 feet high?

6. How many cubic yards of broken stone will be needed to cover a road 300 feet long, 30 feet wide, to the depth of 18 inches?

7. If a cubic foot of water weighs 1000 oz., how many ounces does a gallon weigh, assuming $7\frac{1}{2}$ gallons to the cubic foot?

8. Assuming that zinc is 8 times as heavy as water, how many pounds will a cubic foot of zinc weigh?

9. How many cubic yards of earth will be removed in digging a ditch 300 feet long, 6 feet wide, $4\frac{1}{2}$ feet deep?

10. How many cords of wood are there in a pile 80 feet long, 4 feet wide, 4 feet high?

306. Written Problems.

NOTE. 150' is read 150 feet, 18'' is read 18 inches; $8'' \times 4'' \times 2''$ is read 8 inches by 4 inches by 2 inches.

1. Find the number of cubic yards in a stone wall 150' long, 6' high, 18'' thick.

2. How many perches of stone, of $24\frac{3}{4}$ cu. ft. each, are there in a wall 8 rods long, 6 feet high, and 3 feet thick?

3. How many bricks $8'' \times 4'' \times 2\frac{1}{2}''$ will be required for a wall 300' long, 6' high, and 18'' thick, no allowance being made for mortar?

$$\text{Number of bricks} = \frac{3600 (') \times 72 (') \times 18 (')}{8 (') \times 4 (') \times 2\frac{1}{2} (')} \quad \text{Cancel.}$$

4. How many bricks will be contained in each side wall of a house 60 feet deep and 40 feet high, the wall being 12 inches thick, assuming that 22 bricks with mortar will make a cubic foot?

5. A bin is 9 ft. 4 in. long, 5 ft. 4 in. wide, and 2 ft. high. How many bushels of 2150.4 cu. in. each will it contain?

6. Find the capacity in gallons (231 cu. in.) of a tank measuring 7 ft. 4 in. by 4 ft. 8 in. by 2 ft.

7. How many cords are there in a pile of wood 168 ft. long, 16 ft. high, 4 ft. wide? Find its value at \$4 per cord.

8. A block of dressed marble measures 6 ft. by 4 ft. by 2 ft. What is its weight, if marble is 2.84 times as heavy as water?

9. How many cubic yards of earth will be removed in making an excavation 24 ft. wide, 54 ft. long, and 12 ft. deep? Find the cost of hauling it at 25¢ per load of a cubic yard, assuming that $\frac{3}{4}$ cu. yd. before digging will make 1 cu. yd. in the wagon.

10. A coal pocket 30 ft. by 14 ft. contains 36 tons of coal. How high does the coal stand in the pocket, if a ton contains 35 cubic feet?

BOARD MEASURE.

307. In the measurement of lumber, the unit is the *board foot*. A board foot is 1 ft. long, 1 ft. wide, and 1 in. thick. The number of board feet contained in a board 1 in. thick is equal to the number of square feet that it will cover.

308. *To find the number of board feet in lumber, multiply the length in feet by the width in feet by the thickness in inches.*

NOTE. Boards less than an inch thick are considered as having a thickness of an inch.

309. Oral Exercises.

Find the number of board feet in each of the following planks, joists, sills, etc.:

1. 16' long, 6" wide, 6" thick.

2. 16' long, 6" wide, $\frac{3}{4}$ " thick.

3. 16' long, 6" wide, $\frac{1}{2}$ " thick.
4. 16' long, 9" wide, 2" thick.
5. 16' long, 12" wide, 3" thick.
6. 18' long, 3" wide, 2" thick.
7. 14' long, 6" wide, 2" thick.
8. 12' long, 8" wide, 4" thick.
9. 12' long, 9" wide, 2" thick.
10. 12' long, 8" wide, 3" thick.

310. Written Exercises.

1. Find the cost of 120 boards 16' long, 10" wide, 1" thick at \$35 per M.

2. How many board feet in 20 3-inch planks, each 15 ft. long and 10 in. wide?

3. The sills for a house consist of 160 running feet 8" wide, 8" thick. Find the cost at \$50 per M.

4. Find the cost of fence material needed to inclose a field 32 rd. \times 40 rd. with a tight board fence 6 ft. high. The boards, costing \$14 per M, are nailed to posts placed 6 ft. apart, which cost \$8.75 per hundred.

5. What will be the cost of a fence 6 boards high inclosing a garden 512 feet long and 256 feet wide, the base board being 12" wide, the top board 6" wide, and the others 8" wide? The posts 8 feet apart, cost 15 cents each, the boards cost \$48 per M, and labor, nails, etc., cost \$25.

6. For the framework of a building the following materials are needed:

- Sills, 8" \times 3", 120 running feet.
- Joists, 10" \times 3", 900 running feet.
- Beams, 8" \times 2", 120 running feet.
- Posts, 5" \times 5", 120 running feet.

Rafters, $6'' \times 2''$, 640 running feet.

Sheathing, $3'' \times 2''$, 800 running feet.

Find the cost at \$20 per M.

7. Find the cost of the boards required for 1600 square feet of flooring at \$28 per M, adding $\frac{1}{5}$ for waste in matching.

8. At \$1 per bundle of 250 shingles, what will be the cost of the shingles for a double roof, each half measuring 50' by 25', assuming that a shingle covers 5" by 4", and adding $\frac{1}{12}$ of the number for waste?

9. Find the cost at 15 cents a square foot of dressing 6 stone steps on the top, one side, and both ends, each step being 8 ft. by 2 ft. by 3 ft.

10. How many board feet will be contained in 6 wooden pillars 9 feet high and 18 inches square?

PRACTICAL APPLICATIONS.

MASONRY AND BRICKWORK.

311. In calculating the cubical contents of walls for the purpose of ascertaining the quantity of materials needed, openings are deducted and allowance is made for mortar.

312. When masons contract to build walls by the cubic foot or the perch, it is their practice to calculate the contents of the walls inclosing a cellar, for instance, by measuring only the outside, the extra number of cubic yards being considered a compensation for the additional time spent on the corners. For openings in a wall, the mason usually deducts one half of the space; this allowance for openings should be fixed in the agreement.

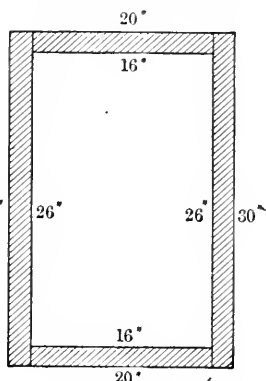
313. Written Exercises.

1. Find the number of square feet of ground covered by a two-foot wall, surrounding a plot 26 ft. by 16 ft.

The outside measurements of the plot are 30 ft. (26 ft. + 2 ft. + 2 ft.) by 20 ft. (16 ft. + 2 ft. + 2 ft.). This makes the area of the whole plot 600 square feet. The area of the inside plot is (26×16) sq. ft., or 416 sq. ft. The area covered by the wall is 600 sq. ft. - 416 sq. ft., or 184 sq. ft.

The area of the ground covered by the wall may also be found as follows :

2 sides, each 30 ft. long, 2 ft. wide	120 sq. ft.
2 sides, each 16 ft. long, 2 ft. wide	64 sq. ft.
Total	184 sq. ft.



2. How many perches of masonry in a wall 9 ft. high and 2 ft. thick surrounding a plot 26 ft. by 16 ft.?

A perch contains $24\frac{3}{4}$ cu. ft. Number of perches = $(184 \times 9) \div 24\frac{3}{4}$.

3. The four walls of a cellar, measuring 30 ft. by 20 ft. on the outside of the walls, are 9 ft. high and 2 ft. thick. How many cubic feet of masonry will they contain, deducting one half for a door $8' \times 4'$ and 4 windows, each $4' \times 2'$?

4. How many cubic yards of stone in the walls of a cellar measuring $26' \times 16'$ on the inside, the walls being 9 ft. high and 18 in. thick, and containing 1 opening $8 \text{ ft.} \times 4 \text{ ft.}$, and 4 openings, each $4 \text{ ft.} \times 2 \text{ ft.}$, deducting $\frac{1}{5}$ for mortar and other filling?

5. Find the cost, at \$5 per perch, to build the walls of a cellar $30' \times 20'$, outside measurement, the walls to be 9 ft. high and 2 ft. thick, deducting one half for an opening $8' \times 4'$, and for four others, each $4' \times 2'$.

6. At 22 bricks to the cubic foot, how many bricks will be needed for the walls of a cellar measuring $30' \times 20'$ on the outside, the walls to be 12" thick and 9' high? Deduct one half for a door $8' \times 4'$, and for 4 windows, each $4' \times 2'$.

7. How many cubic feet of concrete will be required for a floor 4" thick in a cellar measuring $28'$ by $18'$? The con-

crete of this floor is composed of a 3-inch layer, of which $\frac{1}{7}$ is ordinary cement, $\frac{1}{7}$ sand, and $\frac{5}{7}$ broken stone. The top layer, 1 inch, is composed of equal parts of sand and Portland cement. How many cubic feet of each will be required?

8. How many cubic yards of stone will be used in building a wall 100 ft. long, 6 ft. high, and 3 ft. thick, deducting $\frac{1}{3}$ for mortar? If the mortar is $\frac{4}{5}$ sand and $\frac{1}{5}$ lime, how many cubic yards of sand will be needed, and how many bushels of lime at $\frac{4}{5}$ bushel to the cubic foot?

PAINTING AND PLASTERING.

314. In calculating the materials required in painting or plastering a room, deduction is made for all openings.

315. In ascertaining the cost of labor, however, due allowance must be made for the extra time required in working on a wall containing windows, doors, and other openings. For instance, a man could kalsomine an unbroken wall in perhaps less time than he would require to complete a wall of the same dimensions containing several windows. In such a case, he would be unwilling to make any allowance for openings.

316. Wooden laths are usually sold in bundles of 50 or 100 each. They are 4 feet long and generally $1\frac{1}{2}$ inches wide. Fifteen laths of this width will be required for a square yard, the laths being laid about $\frac{3}{8}$ inch apart to permit the first coat of plaster to pass between them and thus to obtain a better hold. Metallic lathing of various kinds is in use. It is a protection against fire, and its meshes insure a better hold for the plaster, especially in ceilings.

317. Plastering is usually applied in three coats. In the first, called the "scratch" coat, the plaster is generally mixed with cattle hair, to render it less liable to crumble.

The second coat is called the "brown" coat, and the last is called the "white" coat. Patent plasters which require only two coats and need no hair, are sometimes employed.

318. Outside painting requires at least two coats for new work. Ready-mixed paints of all colors and of all qualities are easily procurable. The owner of a building needs to calculate only the quantity required.

319. Written Exercises.

1. How many bundles of 100 laths will be required for 4 bedrooms, each $18' \times 12'$, and 9' high, deducting 56 sq. ft. for openings in each room, and allowing 18 laths to the square yard, including waste?

2. Find the cost of the laths at 40 ¢ per bundle, and the cost of the labor at 6 ¢ per square yard, deducting only one half for the openings.

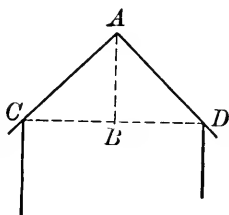
3. How much will be paid for plastering these rooms at 30 ¢ per square yard for the material, and an average of 5 ¢ per square yard for each of three coats, making full deduction for openings in ascertaining cost of material and a deduction of one half in determining the sum paid for labor?

4. What does a contractor receive at 45 ¢ per square yard for plastering a room 22 ft. long, 16 ft. wide, and 12 ft. high, allowing one half for 2 windows, each $6' \times 3\frac{1}{2}'$, and a door $8' \times 4' 3''$?

5. How much does he pay for labor at \$4 per day, the workman putting on 132 sq. yd. of the first coat per day, $\frac{1}{2}$ as much of the second coat in a day, and $\frac{1}{2}$ as much of the finishing coat in a day? Full allowance for openings.

6. What will be the total cost of painting the floor of the foregoing room at 27 cents per square yard, and of kalsomining the four walls and the ceiling at 18 cents per square yard, no allowance being made for openings?

7. Find the cost of painting the outside of a house at 27¢ per square yard, the dimensions being as follows: each side 48 ft. long, 36 ft. high; the front and the back 24 ft. wide and 24 ft. high to the eaves at *C* and *D*, and 36 ft. high to the ridge at *A*.



One half allowance is made for 12 windows, each $6' \times 3\frac{1}{2}'$, and 3 doors, each $8' \times 4\frac{1}{2}'$.

The area of $ACD = \frac{1}{2} (CD \times AB)$.

8. What will it cost to stain the roof at 18 cents per square yard, each half of the roof measuring 21 feet by 51 feet?

9. How many gallons of paint will be required to give two coats to the house, assuming that a gallon will cover 60 sq. ft. the first coat or 80 sq. ft. the second coat? Make full allowance for openings.

ROOFING AND FLOORING.

320. Shingles are generally 16 inches long and of various widths. They are packed in bundles, each of which equals 250 shingles 4 inches wide; that is, the total width of the shingles in a bundle is 1000 inches. The surface covered by a bundle of shingles depends upon the surface laid "to the weather." If each shingle overlaps 12 inches of the one beneath, the shingles are said to be laid 4 inches to the weather, and a bundle will cover 4000 square inches, making no allowance for waste. Shingles are sold by the thousand, consisting of four bundles.

321. An average roofing slate measures 18 inches by 9 inches. Slates are generally laid 7 inches "to the weather," the surface then covered by a slate being $9'' \times 7''$, or 63 square inches. If they are laid 6 inches to the weather, the surface covered by a slate will be 54 square inches.

322. The unit of measure in roofing is the *square* of 100 sq. ft.

323. In laying floors, the boards are "tongued and grooved," the tongue of one board being fitted into the groove of the next. The tongue generally measures $\frac{1}{2}$ inch, so that a board 3 inches wide will cover only $2\frac{1}{2}$ inches of the width of the floor.

324. The number of board feet required to cover a surface with ordinary boards of 1 inch or less in thickness is equal to the number of square feet in the floor. In buying tongued and grooved boards, a fraction of the number of square feet must be added to find the number of board feet required.

Assuming 12 ft. as the length of a board, and 3 in. as its width before tonguing, the number of board feet is 3. The number of square feet covered by each is $(12 \times 2\frac{1}{2}) \div 12$, or $2\frac{1}{2}$; that is, to cover $2\frac{1}{2}$ sq. ft., 3 sq. ft. must be paid for, or to cover 5 sq. ft., 6 sq. ft. must be paid for. The number of square feet to be paid for is therefore $1\frac{1}{3}$ times the number of square feet to be covered.

As 6 sq. ft. of tongued 1-in. board, 3 in. wide, will cover only 5 sq. ft. of floor, such boards cover only $\frac{5}{6}$ of the number of square feet (or board feet) purchased.

325. Written Exercises.

1. How many boards $2\frac{1}{2}$ in. wide and 12 ft. long will be required for a floor 24 ft. long and 20 ft. wide? If each board has to be paid for as being 3 in. wide, measuring the tongue, how many board feet of floor must be ordered?

Area to be covered, 480 sq. ft.

Area covered by 1 board, $2\frac{1}{2}$ sq. ft.

Number of boards, $480 \div 2\frac{1}{2}$.

2. How many square feet of surface will be covered by 192 boards 12 feet long, 3 inches wide, if $\frac{1}{6}$ of the surface of the boards is deducted for tonguing?

3. Find the number of board feet of flooring that is required for a room $32' \times 27\frac{1}{2}'$, the board being 3 inches wide, including $\frac{1}{2}$ inch of tonguing.

The number of square feet to be covered $= 32 \times 27\frac{1}{2}$.

Width of board less tonguing $= 2\frac{1}{2}'' = \frac{5}{4}''$.

Total length of boards required $= (32 \times 27\frac{1}{2}) \div \frac{5}{4}$.

Find the number of board feet in boards of the required length and 3 inches wide, or $\frac{1}{4}$ foot.

Compare the answer thus obtained with the result obtained by adding to the area of the floor $\frac{1}{3}$ of the area.

4. Find the number of board feet of flooring that is required for a room $32' \times 27\frac{1}{2}'$, the boards to be 6 inches wide, including $\frac{1}{2}$ inch of tonguing.

NOTE. Why does this result differ from the result in the preceding problem?

5. What fraction of a 3-inch board is lost in the tonguing? What fraction of a 6-inch board is lost?

What fraction of the surface of a floor must be added when 3-inch boards are used? What fraction when 6-inch boards are used?

6. How many bundles of shingles laid 4 inches to the weather will be required to cover a roof, each half of which measures 50 feet by 20 feet?

7. How many days will a man take to shingle a roof 50 feet by 40 feet, if he lays 2000 shingles per day?

8. Find the cost of slating a roof $56' \times 45'$ at \$16 per square (100 sq. ft.).

9. How many slates are required, if each slate covers $9'' \times 7''$? How many square feet of slates are used, each slate measuring $18'' \times 9''$?

CARPETING AND PAPERING.

326. Ingrain carpet and matting are generally 1 yard wide. Other carpets are $\frac{3}{4}$ yard, or 27 inches, wide. Lino-

leum and oilcloth are made in various widths. All are sold by the running, or linear, yard.

327. Wall paper is sold in rolls. It is usually 18 inches wide and the single rolls are 8 yards long, the double rolls being 16 yards long.

328. Owing to the loss in matching patterns, it is difficult to ascertain the exact quantity of carpet required for a given room.

The difficulty is increased in the case of wall paper, the location of windows, doors, and other openings, frequently necessitating much waste.

329. Written Exercises.

1. How many yards of carpeting 1 yd. wide will be required for the floor of a room 24 ft. long, 18 ft. wide?

If the pattern is repeated at intervals of any aliquot part of a yard, there will be no loss in matching. As the room is 8 yd. long and 6 yd. wide, it will require 8 strips of carpet, each containing 6 yd., or 6 strips, each containing 8 yd. *Ans.* 48 yd.

2. How many yards of carpeting $\frac{3}{4}$ yd. wide will be required for the floor of a room 24 ft. long, 18 ft. wide?

If the strips run the length of the room, each strip will be 8 yd. long. The number of strips is 6 yd. $\div \frac{3}{4}$ yd., or 8. The number of linear yards is 8×8 , or 64. *Ans.* 64 yd.

3. How many yards of carpeting $\frac{3}{4}$ yd. wide will be required for the floor of a room 25 ft. long, 19 ft. wide?

If the strips run the length of the room, each strip will be $8\frac{1}{3}$ yd. long. Dividing 19 ft. by $2\frac{1}{4}$ ft., we get a quotient of 8, and a remainder of 1 ft. It is necessary to buy a ninth strip to obtain the piece 1 ft. wide necessary to complete the carpet. $8\frac{1}{3}$ yd. $\times 9 = 75$ yd. *Ans.* 75 yd.

If the strips run across the room, each strip will be $6\frac{1}{3}$ yd. long. The number of strips required will be $12(25 \div 2\frac{1}{4} = 11+)$. $6\frac{1}{3}$ yd. $\times 12 = 76$ yd. *Ans.* 76 yd.

One yard will be saved by carpeting the room the short way.

4. How many yards of carpet 1 yard wide will be required for the floor of a room 25 feet long, 19 feet wide, if the strips run the length of the room?

5. How many yards of carpet 1 yd. wide will be needed for the floor of the same room, if the strips run the width of the room?

6. How many rolls of paper 8 yards long, 18 inches wide will be required for the ceiling of a room 24 feet long, 18 feet wide?

7. If the foregoing room is 9 ft. high above the base-board and below the border, how many single rolls of paper will be required for the walls, deducting for two windows, each 6 ft. \times $4\frac{1}{2}$ ft., and a door 8 ft. \times 6 ft., no waste arising in matching the pattern, etc.?

8. Find the cost of papering the walls and the ceiling of the foregoing room, at 50 ¢ per roll for the paper, and 10 ¢ per running yard for the border around the four walls, adding \$3 additional for labor, etc.

9. Find the cost of carpeting this room with carpet 27 inches wide, costing \$1.25 per running yard, and 10 cents per square yard for sewing, lining, and laying.

10. The floor of a room 14 ft. by 18 ft. is to be covered with matting 1 yd. wide. Which will be the cheaper way to lay the matting, and how much will be saved if the matting costs 65 ¢ per yard?

11. The floor of a kitchen 24 ft. by $16\frac{1}{2}$ ft. is covered with linoleum 2 yd. wide. Find the cost at \$1 per square yard, the strips running the more economical way.

12. A floor $16' \times 12'$ is to be covered with carpet 27 in. wide, at \$2 per yard, made and laid. What will be the cost, if 9 inches must be cut from the length of each strip, except the first, to match the pattern, the carpet running the length of the room?

THE METRIC SYSTEM.

330. The metric system of weights and measures was devised in France about the beginning of the last century. It has been adopted in many countries, and legalized in others, including the United States. In this country its use is largely limited to the compounding of prescriptions by druggists and to the scientific work of high schools and colleges.

331. The countries that use the metric system in all their transactions find it of great advantage in their exchange of commodities, not being compelled, as we are, in buying goods in France, for instance, to change the meters, liters, and kilograms, into yards, gallons, and pounds, in order to compare prices.

332. The basis of the metric system is the *meter*, which is one ten-millionth part of the quarter meridian passing through Paris. The meter is 39.37 inches, or a little over $1\frac{1}{12}$ yard.

333. The *liter* is the unit of volume. It has the capacity of a cube, each edge of which is a decimeter, or $\frac{1}{10}$ meter. It is a little larger than our liquid quart.

334. The unit of weight is the *gram*. A liter of pure water at a certain temperature weighs 1000 grams, or a kilogram, about $2\frac{1}{5}$ pounds.

335. In changing from one unit to another of the same table, the reduction is effected by the removal of the decimal point or by the annexing of ciphers.

336. In naming the units of this system the following prefixes are used :

deci meaning $\frac{1}{10}$	deka meaning 10
centi meaning $\frac{1}{100}$	hecto meaning 100
milli meaning $\frac{1}{1000}$	kilo meaning 1000
myria meaning 10,000	

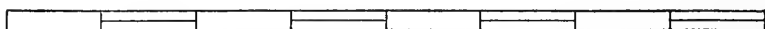
LONG MEASURE.

337. Table.

10 millimeters (^{mm})	= 1 centimeter (^{cm})
10 centimeters	= 1 decimeter (^{dm})
10 decimeters	= 1 meter (^m)
10 meters	= 1 dekameter (^{Dm})
10 dekameters	= 1 hectometer (^{hm})
10 hectometers	= 1 kilometer (^{Km})
10 kilometers	= 1 myriameter (^{Mm})

338. The measures in common use are the *millimeter*, the *centimeter*, the *meter*, and the *kilometer*.

Our nickel 5¢ piece is 2 centimeters in diameter and 2 millimeters thick. An opening through which a nickel will just pass is 2 centimeters long and 2 millimeters wide. The accompanying scale is 8 centimeters long and 2 millimeters wide.



339. The *meter* is employed in denoting short distances, such as we measure in yards. The *centimeter* is employed to express widths of ribbon, dress goods, carpets, boards, etc. The thickness of coins, the diameter of wire, etc., are expressed in *millimeters*. Long distances are given in *kilometers*.

340. Sight Exercises.

1. Change 3576^m into millimeters; into centimeters; into kilometers.

2. How many centimeters in 584^m? How many millimeters? What decimal of a kilometer?

3. Add 42^m and 573^{cm} .
4. From 30^m take 250^{cm} .
5. Multiply 2.6^m by 10.
6. Divide 45^m by 100.
7. How many nickels laid side by side will measure a meter?
8. How many dollars are there in a pile of nickels 1 meter in height?
9. How many strips of carpet, each 75 centimeters wide, will be required to cover a floor 9 meters wide?
10. If a man's step is 75^{cm} , how many steps will he take in going $\frac{3}{5}$ of a kilometer?
11. Assuming a kilometer to be $\frac{5}{8}$ mile, how many kilometers will equal 100 miles?
12. If a boy wears a 12-inch collar, for what size, in centimeters, would he ask in a store in Paris?
13. What is the approximate width in inches of a board 25 centimeters wide?
14. The unit of thickness in French board measure is 25 millimeters. What is the corresponding thickness in inches, assuming the meter to be 40 inches?
15. Assuming the meter as 3 ft. 3 in., how many feet in length is a board 4 meters long?
16. Two towns are 120 kilometers apart. What is the distance between them in miles at $\frac{5}{8}$ mile to the kilometer?

341. Written Exercises.

1. Express the sum of the following in meters:

$$3.46^m + 2^{Km} + 415^{cm} + 18^{mm}.$$
2. From 1 kilometer take 295 centimeters. Give result in meters.

3. How many meters in 75 hundredths of 18.48 kilometers?

4. Give in centimeters the quotient of $84^m \div .75$.

5. How many strips of matting, each 95 centimeters wide, will be required for the floor of a room 11.4 meters wide? If the room is 13.5 meters long, how many linear meters of matting will it take to cover the floor?

6. Find the total cost of the following:

2.30 meters of cloth @ 9.50 francs.

2.50 meters of satin @ 6.40 francs.

4.75 meters of ribbon @ 3.50 francs.

7. To make a dozen napkins, 10.80 meters of linen are required. How many meters are necessary to make 125 napkins?

8. A dealer pays 1836 francs for 3 pieces of cloth costing 8.50 francs per meter. How many meters in each piece?

9. A man receives 4.45 francs per day. How many francs will he receive for digging a ditch 63.45 meters long, if he digs 2.35 meters in a day?

10. A man takes 120 steps of 70 centimeters each in a minute. How many meters will he travel in an hour?

SQUARE MEASURE.

342. Preliminary Exercises.

1. How many square decimeters in a square 10 decimeters long, 10 decimeters wide?

2. How many square meters in a square 1 meter long, 1 meter wide?

3. How many square decimeters in a square meter?

4. How many square centimeters in a square 1 decimeter long and 1 decimeter wide?

5. How many square centimeters in a square decimeter?

343. Table.

100 sq. centimeters (qcm)	= 1 sq. decimeter (qdm)
100 sq. decimeters	= 1 sq. meter (qm)
100 sq. meters	= 1 sq. dekameter (qDm)
100 sq. dekameters	= 1 sq. hektometer
100 sq. hektometers	= 1 sq. kilometer

344. *In calculating areas, express the length and the width in the same **linear unit**. The product of these two numbers will give the area in **square units** of the same name.*

345. Sight Exercises.

1. How many sq. decimeters in a square 11 decimeters long, 11 decimeters wide?
2. How many sq. meters in a square $1.1^{\text{m}} \times 1.1^{\text{m}}$?
3. Change 121 sq. decimeters to sq. meters. How many decimal places are pointed off?
4. Change 121 decimeters to meters. How many decimal places are pointed off?
5. How many square meters in a rectangle $44^{\text{m}} \times 25^{\text{m}}$?
6. How many square dekameters in a rectangle $4.4^{\text{Dm}} \times 2.5^{\text{Dm}}$?
7. Change 1100 square meters to square dekameters.
8. Change 11 square dekameters to square meters.
9. How many square meters in the floor of a room 8 meters long, 5.5 meters wide?
10. How many square meters in the surface of a board 4 meters long, 25 centimeters wide?

DRY AND LIQUID MEASURE.

346. Table.

10 deciliters (dl)	= 1 liter (l)
10 liters	= 1 dekaliter (Di)
10 dekaliters	= 1 hektoliter (Hl)

347. A *liter* is the equivalent of a hollow cube 1 decimeter long, 1 decimeter wide, 1 decimeter high. The *liter* is the unit employed in buying and selling all liquids and grains that are measured. Large quantities are expressed in *hektoliters*. All the other prefixes can be employed, but those given in the table are the most common.

348. *To ascertain the contents of a rectangular vessel in liters, find the product of the three dimensions, each expressed in decimeters.*

349. Written Exercises.

1. How many hektoliters of grain will a bin contain 40^m long, 2^m wide, 1.2^m deep?

2. A tank measures 4^m \times 2^m \times 1.2^m. How many liters of oil will it hold?

3. A piece of ground 20^m \times 10^m produced 5^{Pl} 2^l of beans. What is the yield of a square meter in deciliters?

4. A field produces 648 sheaves of wheat, 4 sheaves yielding a double dekaliter of grain. How many hektoliters in the crop?

5. A pipe discharges 28 liters of water in 5 seconds. How many hektoliters are discharged in 24 hours?

350. Table of Weight.

10 milligrams (mg)	= 1 centigram
10 centigrams	= 1 decigram
10 decigrams	= 1 gram (g)
10 grams	= 1 dekagram
10 dekagrams	= 1 hektogram
10 hektograms	= 1 kilogram (Kg)
1000 kilos	= 1 metric ton (t)

351. A cubic centimeter of water weighs 1 *gram*. A liter of water weighs 1000 grams, called a *kilogram* or *kilo*. A cubic meter of water weighs a *metric ton*.

352. Sight Exercises.

1. A vessel measures $3^{\text{dm}} \times 3^{\text{dm}} \times 3^{\text{dm}}$. How many liters will it contain?

2. Find the weight in kilos of the water that can be contained in a cubical measure $3^{\text{dm}} \times 3^{\text{dm}} \times 3^{\text{dm}}$.

3. If cast iron is 7.5 times as heavy as water, how many kilos will an iron cube weigh, each side of which measures 2^{dm} ?

4. Find the weight in metric tons of the material removed in excavating a cellar $8^{\text{m}} \times 5^{\text{m}} \times 3^{\text{m}}$, assuming that it is twice as heavy as water.

5. Mercury is 13.6 times as heavy as water. How many kilos does a liter of mercury weigh?

6. How many kilos in a cubical block of marble, each side of which measures 1^{m} , marble being 2.837 times as heavy as water?

7. An empty pail weighing 2 kilos is partly filled with water. How many liters of water does it contain, if the weight of the pail and the water is 10 kilos?

8. A 5¢ nickel piece weighs 5 grams. How many dollars in nickels will weigh a kilo?

9. Find the weight of a double dekaliter of wheat at 75 kilos per hektoliter.

10. A franc contains 100 centimes. Bronze coins of 5 and 10 centimes weigh 5 and 10 grams respectively. What is the value in francs of a kilo of bronze coins?

353. Written Exercises.

1. What is the weight of the hay consumed by a horse in 30 days, if he eats $7^{\text{Kg}} 5^{\text{Hg}}$ per day?

2. What is the price of a liter of olive oil weighing 920 grams at $2\frac{1}{2}$ francs per kilo?

3. The front wheels of a carriage are 3.25^m in circumference. How many revolutions will they make in going 6.5^{km} ?
4. A train runs 252.9 kilometers in 5 hours. What is its speed in meters per minute?
5. An empty vase weighs 2947 grams; filled with olive oil, it weighs 15,757 grams. What is the capacity of the vase, if a liter of oil weighs 915 grams? How much would it weigh if filled with water?

354.

COMPARISON OF UNITS.

Length	{	Meter,	39.37 in.	Yard,	.9144 meter
		Kilometer,	.62137 mi.		Mile, 1.6093 kilometers
Capacity	Liter {	1.0567 liquid qt.		Liquid qt., .9463 liter	
		.908 dry qt.		Dry qt., 1.101 liters	
Weight	{	Gram	15.432 gr. troy	Troy gr. 6.48 milligrams	
		Kilo	2.2046 lb. avoird.	Avoird. lb. .4536 kilo	

355. Sight Exercises.

1. How many meters in 1000 yd.?
2. How many miles in 100 kilometers?
3. How many quarts, liquid measure, in 100 liters?
4. How many liters in 100 quarts, dry measure?
5. How many troy grains in a kilo?
6. How many kilos in 1000 pounds avoirdupois?

CHAPTER VI.

RATIO AND PROPORTION; POWERS AND ROOTS.

ARITHMETICAL ANALYSIS.

356. Sight Exercises.

1. If 3 sheep cost $\$12\frac{3}{4}$, what will 8 cost?
2. At the rate of $\$4$ for 25 pounds of cheese, how many pounds can I buy for $\$1.76$?
3. If 7 men can do a piece of work in 12 days, how long would it take 4 men to do it?
4. What will be the cost of $3\frac{1}{4}$ yards of dress goods at the rate of 16 cents per half yard?
5. A garrison requires 140 barrels of flour every 2 weeks. Find the time that 490 barrels will last.

Each of the foregoing examples is most readily solved "mentally" by the method of *unitary analysis*, as it is sometimes called. In No. 1, the cost of 1 sheep is first found; in No. 2, the cost of 1 pound; in No. 3, the time required by 1 man, etc.

In the next five exercises, the *ratio method* is preferable.

6. Find the cost of 33 sheep at the rate of $\$13$ for 3 sheep. (*Ans.* 11 times $\$13$. Why?)
7. At $\$22$ per dozen, what should be paid for 36 hats? (How many times $\$22$?)
8. If 7 men can do a piece of work in 150 days, how long would it require 21 men to do the same work? (What fraction of 150 days?)
9. Find the cost of 3 yards of merino at the rate of 22ℓ for $\frac{3}{4}$ yd. (How many times 22ℓ ?)
10. If a garrison uses 111 barrels of flour every 4 weeks, how long will 333 barrels last?

11. When $\frac{7}{8}$ lb. butter costs 21¢, find the cost of $1\frac{1}{2}$ lb.

Oral problems involving the comparison of two fractional quantities may be solved by giving the fractions the same denominator.

Since $1\frac{1}{2}$, or $\frac{3}{2}$, equals $\frac{12}{8}$, the foregoing problem requires the cost of 12 eighths when 7 eighths cost 21¢. The comparison may now be made between 12 and 7 as follows, the *eighths* being ignored:

When 7 parts cost 21¢, 1 part costs 3¢, and 12 parts cost 36¢.
Ans.

Solve the next five problems by reducing the fractions to a common denominator.

12. I pay \$8000 for 2 thirds of a farm. What should I pay for 3 fourths of it at the same rate?

13. A man sells a cow for $\frac{4}{5}$ of its cost, receiving for it \$48. What would he have received, if he had sold the cow for $\frac{5}{6}$ of its cost?

14. A roll of matting $\frac{3}{4}$ yd. wide contains 27 sq. yd. How many square yards in another roll of the same length, but $1\frac{1}{9}$ yd. wide?

15. If $\frac{3}{7}$ of a number is $24\frac{4}{5}$, what is $\frac{3}{4}$ of the same number?

16. A person owning $\frac{2}{11}$ of a business receives \$4000 as his share of the profits. How much should the owner of $\frac{3}{2}$ of the business receive?

17. At 3 pairs for 50 cents, what will be the cost of $1\frac{1}{4}$ dozen pairs?

18. A wagon load of sand containing 2 cubic yards costs \$1.50. What is the cost of 28 cubic yards?

19. How long will a supply of provisions last 75 men, if the same quantity will last 25 men $7\frac{1}{2}$ months?

20. A field of grass can be mowed by 8 men in $4\frac{1}{2}$ days. How many men will be required to mow it in 6 days?

21. By hauling hay to market in loads of $\frac{3}{4}$ ton each, 72 trips are required. How many trips will be saved, when the road is so improved that a load of $1\frac{1}{8}$ tons can be hauled?

22. Lumber for a building can be hauled in 16 days, if the teams haul 3 loads per day. How many days will be required if the teams haul 4 loads per day?

23. A man has sufficient hay to last his 8 horses for 12 weeks. How long will it last if he sells 2 of his horses?

24. There are required for a barn floor 90 boards 14 inches wide. How many will be needed, if 9-inch boards are used?

25. The captain of a vessel has 60 days' provisions for 36 passengers. How many additional passengers can he take, if the voyage lasts but 48 days?

357. Written Problems.

1. Find the cost of 96 bbl. flour, when 34 bbl. cost \$199.75.

Instead of finding the cost of 1 bbl. by dividing \$199.75 by 34 and multiplying the quotient by 96, it will generally be found more satisfactory to leave division for the last operation. The cost of 1 bbl. is indicated $\frac{\$199.75 \times 96}{34}$. Cancel by writing 34 as a divisor, and the cost of 96 is indicated by writing 96 as a multiplier.

2. When the lamps in a factory burn 2 hr. 40 min. per day, a barrel of oil lasts 30 da. How long will it last when the lamps burn 3 hr. 20 min. per day?

Change the time to 160 min. and 200 min.

When the lamps burn 160 min. per day, the oil lasts 30 da.; when they burn 200 min., the oil will last 160 times as long (write 160 in the numerator); when they burn 200 min. per day, the oil will last $\frac{160}{200}$ of the time it would last if the lamps burned 1 min. per day. (Write 200 in the denominator.)

3. A farmer sells $\frac{9}{16}$ of his farm for \$2362.50. What is the value of $\frac{6}{7}$ of the farm at the same rate?

In written examples it is often unnecessary to change the fractions to those having a common denominator.

As $\frac{9}{16}$ of the farm is worth a certain sum, the farm is worth this sum divided by $\frac{9}{16}$, and $\frac{6}{7}$ of the farm is worth the value of the farm multiplied by $\frac{7}{6}$. These fractions can first be written above and below the line, respectively, in the manner shown above, and then rewritten, $\frac{9}{16}$ being inverted, and the denominator being written above the line and the numerator below.

$$\frac{\$2362.50 \times \frac{6}{7}}{\frac{9}{16}}$$

$$\frac{\$2362.50 \times 16 \times 6}{9 \times 7} \quad \text{Cancel.}$$

4. If a bag containing $24\frac{1}{2}$ pounds of flour will make $29\frac{1}{2}$ pounds of bread, how many pounds of bread can be made from a barrel of flour, 196 pounds?

5. If 20° on the equator measures 1383.2 miles, what is the equatorial circumference of the earth?

6. A farmer has sufficient hay to last 120 sheep 36 days. How long would it last 96 sheep? 160 sheep?

7. How many sheep could be fed for 32 days with a supply of hay that would last 120 sheep 36 days?

8. To empty a pond by a pump discharging 28 gallons of water per minute requires 15 days. How long would it take a pump that discharges 35 gallons per minute?

9. A man on a journey finds that at \$8 per day for expenses he can travel only 45 days longer. How many days can he prolong his trip by reducing his expenses by one tenth?

10. If a traveler has sufficient money to last him 110 days, how many days will his trip be shortened if he increases his daily expenses one tenth?

11. A farmer has a quantity of hay sufficient to last his cattle a certain time. By what fraction is the time diminished, if he increases his herd $\frac{1}{10}$? By what fraction is the time increased, if he diminishes his herd $\frac{1}{10}$?

12. If I pay \$120 for the use of a piece of land for 2 yr. 6 mo., what should I pay for its use for 3 yr. 4 mo.?

13. A loans B a certain sum with the understanding that he is to receive \$160 for its use for 3 yr. 4 mo. If B returns it in 2 yr. 6 mo., what should he pay for its use?

14. If 6 men can do a certain piece of work in 24 days, how long will it take 4 men and 4 boys to do it, if a boy does one half as much work as a man?

This method of solving problems by *analysis* is also employed when the problems contain a large number of conditions, but involve only multiplication and division.

15. If 30 men in 40 da., working 10 hr. a day, dig a ditch 450 ft. long, 6 ft. wide, and 4 ft. deep, how long a ditch 8 ft. wide and 3 ft. deep can 50 men dig in 60 da. working 8 hr. a day?

Men	Da.	Hr.	Ft. W.	Ft. D.	
30	40	10	6	4	450 ft. _____ (a)
1	1	1	1	1	
50	60	8	8	3	

Arrange the work as shown in the first horizontal row, each number having its designation written above it. Write 450 ft. last, with a line drawn under it. In the second row place 1 in the column of *men* and reason thus:

“If 30 men dig 450 ft., 1 man will dig $\frac{1}{30}$ 450 ft.,” $\frac{450 \text{ ft.} \times 6 \times 4}{30 \times 40 \times 10}$ (b)

and write 30 in the denominator. Next place 1 in the column of *days*, and reason thus:

“Working 1 da., he will do $\frac{1}{40}$ as much, etc.,” and write 40 in the denominator. Then place 1 in the column of *hours*, and reason:

“Working 1 hr. per day, he will do $\frac{1}{10}$, etc.,” and write 10 in the denominator. Place 1 in the column of *feet wide*, and reason:

“When the ditch is 1 ft. wide, he can do 6 times as many feet of length,”

and write 6 in the numerator. Place 1 in the column of *feet deep*, and reason:

“When the ditch is 1 ft. deep, he can do 4 times as many feet of length.”

and write 4 in the numerator.

The next step is to compare successively 50 men, 60 da., 8 hr., 8 ft. wide, 3 ft. deep, with 1 man, 1 da., 1 hr., 1 ft. wide, 1 ft. deep. Placing 50 in the column of *men*, reason :

“ 50 men will dig 50 times as long a ditch as 1 man,”

$$\frac{450 \text{ ft.} \times 6 \times 4 \times 50 \times 60 \times 8}{30 \times 40 \times 10 \times 8 \times 3} \quad (c)$$

and write 50 in the nu-

merator. Placing 60 in the column of *days*, reason :

“ In 60 da. they will dig a ditch 60 times as long,”

and write 60 in the numerator. Placing 8 in the column of *hours*, reason :

“ Working 8 hr. a day, the length will be 8 times as great,”

and write 8 in the numerator. Placing 8 in the column of *feet wide*, reason :

“ When the ditch is 8 ft. wide, it will be $\frac{1}{8}$ as long,”

and write 8 in the denominator. Placing 3 in the column of *feet deep*, reason :

“ When the ditch is 3 ft. deep, it will be $\frac{1}{3}$ as long.”

and write 3 in the denominator.

The operations indicated partially at (b) and completely at (c) should all be written at (a), the other two being introduced merely to show how the work proceeds.

Find the result.

16. If 8 horses consume 24 bushels of oats in 16 days, how long will 352 bushels last 11 horses ?

17. If a piece of ground 160 rods long and $30\frac{1}{4}$ rods wide yields 15 bushels of wheat, what should be the yield of a piece 121 rods long and 80 rods wide ?

18. If 12 men in 9 days build a wall $67\frac{1}{2}$ feet long when the wall is 8 inches thick, how many men would be required to build $112\frac{1}{2}$ feet of wall in 5 days, the wall to be 12 inches thick ?

19. If 256,000 bricks, each 8 in. \times 4 in. \times 2 in., are required to build a wall, how many concrete blocks, each 4 ft. \times 2 ft. \times 1 ft., would be required for the same wall, no allowance being made for mortar in either case ?

20. If 17 men in 12 days earn \$612, how much should be the earnings of 11 men in 14 days ?

21. If it costs \$18 to thresh the wheat raised in a field 60 rods wide and 80 rods long, what should it cost to thresh the wheat raised in a field 176 rods long, 110 rods wide ?

22. A farmer paid \$140 for material required to fence a field 80 rods long and 60 rods wide. What would be the cost of the material required to inclose a field 176 rods by 110 rods ?

Note the difference between this problem and the preceding one. The yield of the field in No. 21 depends upon an area 60 rd. by 80 rd. ; the cost of the fence in No. 22 depends upon its length, 60 rd. + 80 rd. + 60 rd. + 80 rd. In problem 21, the results are based upon the respective *products* of 60×80 and 110×176 ; in problem 22, these numbers are *addends*, and not *factors*.

23. If it costs \$180 to cement the sides and the bottom of a tank 24 ft. long, 12 ft. wide, 8 ft. deep, what will be the cost of cementing the sides and the bottom of a tank 48 ft. by 24 ft. by 16 ft. ?

24. If it requires 12 hr. 20 min. for a pipe to fill a tank $24' \times 12' \times 8'$, how long will be required for the same pipe to fill a tank $48' \times 24' \times 16'$?

25. If 8 horses use $52\frac{1}{2}$ bushels of oats in 5 weeks, how many bushels will 16 horses use during April, May, and June ?

ANALYSIS BY ALIQUOT PARTS.

358. Business men, as a rule, dislike long operations in multiplication and division. Many problems usually solved by analysis or by proportion, they work by aliquot parts.

359. Written Exercises.

1. If 24 men receive weekly wages of \$316.80, what should be paid 30 men ? 56 men ?

24 men receive	\$316.80	
6 men receive	<u>78.20</u>	$\frac{1}{4}$ of 24 men's wages
30 men receive	\$395.00	Ans.

24 men receive	\$316.80	
24 men receive	316.80	
<u>8 men receive</u>	<u>105.60</u>	$\frac{1}{3}$ of 24 men's wages

2. The keeper of a livery stable uses on an average 1080 lb. hay weekly for 15 horses. How much will 18 horses use per week? 12 horses? 20 horses?

Add $\frac{1}{3}$; subtract $\frac{1}{3}$; add $\frac{1}{3}$.

3. Find the cost of 40 articles at \$29.70 per gross (144).

<u>144 cost \$29.70</u>			or	<u>144 cost \$29.70</u>		
48 cost	\$9.90	$\frac{1}{3}$ gross		36 cost	\$7.425	$\frac{1}{4}$ gross
Deduct	<u>8 cost 1.65</u>	$\frac{1}{6}$ of 48	Add	<u>4 cost .825</u>		$\frac{1}{4}$ of 36

4. If $\frac{3}{4}$ yd. cloth costs \$1.17, what is the cost of a yard?

3 fourths costs \$1.17

1 fourth costs .39 1 fourth is $\frac{1}{3}$ of 3 fourths.

5. A man buys a $\frac{3}{4}$ interest in a business for \$2790. What is the value of the whole business? Of $\frac{7}{8}$ of the business? Of $\frac{2}{3}$?

$\frac{3}{4}$ is worth \$2790	$\frac{6}{8}$ is worth \$2790	$(\frac{3}{4}) \frac{9}{12}$ is worth \$2790
$\frac{1}{4}$ is worth <u> </u>	$\frac{1}{8}$ is worth <u> </u>	$(\frac{1}{6}) \frac{1}{12}$ is worth <u> </u>
1 is worth <u> </u>	$\frac{7}{8}$ is worth <u> </u>	$(\frac{2}{3}) \frac{8}{12}$ is worth <u> </u>

6. If a $\frac{1}{4}$ interest in a factory is worth \$8750 dollars, how much is a $\frac{3}{8}$ interest worth? A $\frac{5}{16}$ interest?

7. After spending $\frac{1}{4}$ of his money, a man has \$84.69 remaining. What had he at first?

8. What is the rent of a house for 3 yr. 5 mo. 18 da. at \$420 per year?

Rent for 1 yr.	\$420	
Rent for 2 yr.		
Rent for 4 mo.		$\frac{1}{3}$ of \$420
Rent for 1 mo.		$\frac{1}{4}$ of 4 mo.
Rent for 15 da.		$\frac{1}{2}$ of 1 mo.
Rent for 3 da.		$\frac{1}{5}$ of 15 da.
Rent for 3 yr. 5 mo. 15 da.	\$	<u> </u>

9. Find the rent of a farm for 2 yr. 7 mo. 20 da. at \$420 per year.

10. A man paid \$1543.50 for calico at the rate of 6¢ per yard. He sold it for 7¢ per yard. What did he receive for it?

$$\begin{array}{rcl} \text{Cost @ 6¢} & \$1543.50 & \\ \text{Profit @ 1¢} & \underline{\hspace{1cm}} & \frac{1}{6} \end{array}$$

11. Calico which cost $4\frac{1}{2}$ ¢ to make was sold by the manufacturer at 6¢ per yard. What was his profit on a sale of \$1543.50? What was the cost of manufacturing these goods?

12. A quantity of goods cost \$1543.50 at 6¢ per pound; find the amount received for them at 5¢ per pound. At $5\frac{1}{2}$ ¢ per pound. At 4¢ per pound.

13. A lot of coffee weighing 4864 lb. was sold at 24¢ per pound. What was received for it?

$$\begin{array}{rcl} \text{Cost of 4864 lb. @ 25¢} & = \frac{1}{4} \text{ of } \$4864 & = \$1216. \\ \text{Deduct cost of 4864 lb. @ 1¢} & & = \underline{\hspace{1cm}} \\ \text{Cost of 4864 lb. @ 24¢} & & \end{array}$$

14. Find the cost of 4864 lb. coffee :

$$\begin{array}{ll} (a) \text{ At } 24\frac{1}{2}\text{¢.} & (c) \text{ At } 24\frac{3}{4}\text{¢.} \\ (b) \text{ At } 24\frac{7}{8}\text{¢.} & (d) \text{ At } 24\frac{5}{6}\text{¢.} \end{array}$$

15. Find the cost of 3248 yards :

$$\begin{array}{ll} (a) \text{ At } 12\frac{1}{2}\text{¢.} & (d) \text{ At } 13\frac{3}{4}\text{¢.} \\ (b) \text{ At } 13\frac{1}{4}\text{¢.} & (e) \text{ At } 12\text{¢.} \\ (c) \text{ At } 13\frac{1}{2}\text{¢.} & (f) \text{ At } 11\frac{1}{2}\text{¢.} \end{array}$$

16. Find the cost of 8576 gallons :

$$\begin{array}{ll} (a) \text{ At } 99\frac{1}{2}\text{¢.} & (e) \text{ At } 87\frac{5}{8}\text{¢.} \\ (b) \text{ At } 49\frac{1}{2}\text{¢.} & (f) \text{ At } 99\frac{3}{4}\text{¢.} \\ (c) \text{ At } 32\frac{1}{2}\text{¢.} & (g) \text{ At } 88\frac{1}{2}\text{¢.} \\ (d) \text{ At } 19\frac{7}{8}\text{¢.} & (h) \text{ At } 86\frac{1}{2}\text{¢.} \end{array}$$

RATIO.

360. When we speak of the **ratio** of two numbers or quantities, we mean the comparison of two numbers by dividing the first by the second.

361. The *sign of ratio* is the colon ($:$) placed between the numbers to be compared.

To express the ratio of 4 to 12, we write $4:12$. This means $4 \div 12$, or $\frac{4}{12}$, which is equal to $\frac{1}{3}$.

In Continental Europe the colon ($:$) is the sign both of ratio and of division, our sign (\div) not being employed.

362. The numbers compared constitute a *couplet*, the first term of which is called the *antecedent*, and the second the *consequent*.

363. Only like numbers can be compared. Thus, the ratio of 6 pecks to 3 bushels is obtained by changing the bushels to pecks, so that the couplet becomes 6 pecks to 12 pecks.

The ratio is $\frac{6 \text{ pecks}}{12 \text{ pecks}} = \frac{6}{12} = \frac{1}{2}$.

364. Fractions may be compared by reducing them to a common denominator and comparing their numerators. Thus, the ratio of $\frac{3}{4}$ to $\frac{2}{3}$ is the ratio of $\frac{15}{12}$ to $\frac{8}{12}$, or 15 to 8, the result being $15 \div 8$, or $1\frac{7}{8}$.

365. Sight Exercises.

Find the ratios of:

1. 18 to 9.

6. 51 ft. to 17 ft.

2. 12 bu. to 4 bu.

7. 45 to 90.

3. \$4.50 to \$15.

8. 28 in. to 21 in.

4. \$14 to \$2.

9. 3 tenths to 4 fifths.

5. 15 mi. to 105 mi.

10. 8 in. to 2 ft.

11. What is the ratio of 2 quarts to a bushel?
12. John earns \$1.50 per day and Henry earns \$2 per day. What is the ratio of their respective earnings?
13. Tea costs 60¢ per pound and coffee 35¢. Find the ratio of their respective prices.
14. A earns \$9 in the same time that B earns \$12. What is the ratio of A's wages to B's?
15. A man's salary is \$1500 a year and his expenses are \$900. Find the ratio of his expenses to his salary.
16. A boy's age is 15 and his father's is 45.
 - (a) What is the ratio of the son's age to that of his father?
 - (b) What was it 5 years ago?
 - (c) What was the ratio ten years ago?
 - (d) What will it be in 5 years?
 - (e) What will the ratio be in 15 years?
17. B's age is to A's as 5 to 3. How old is A if B's age is 20 years?
18. Give 3 couplets whose ratio is 1 : 5.
19. Give 3 couplets whose ratio is 5 to 2.

INVERSE RATIO.

366 The ratio heretofore considered is called **direct ratio**. The ratio of 18 men to 6 men is 3, as far as wages are concerned or the amount of work done; that is, 18 men will do 3 times as much work as 6 men, or 18 men will earn 3 times as much money as 6 men. If, however, the problem considers the time taken by 18 men to do a certain work as compared with the time taken by 6 men, the ratio is $\frac{1}{3}$. This is called **inverse ratio**.

367. Oral Exercises.

1. What is the ratio of 6 horses to 9 horses as to the amount of work done? As to the time a certain quantity of oats will last?

2. What is the ratio of 4¢ muslin to 5¢ muslin as to the quantity that can be bought for a certain sum? As to the cost of a certain number of yards?

3. One wheel is 9 feet in circumference and another is 12 feet. Find the ratio of the distances covered in the same number of revolutions of each; the ratio of the number of revolutions made by each in going the same distance.

4. Find the ratio between the quantities of tea and coffee, respectively, that can be purchased for \$4.20, the former being worth 60¢ per pound and the latter 35¢.

5. What is the ratio of the quantity of 60¢ material and the quantity of 35¢ material that can be purchased for a dollar?

PROPORTION

368. Two couplets having equal ratios may form a proportion.

369. The *sign of proportion* is the double colon (::) placed between two ratios. The sign of equality (=) is also employed.

370. Since 4:8 is equal to 9:18, the ratio in each case being $\frac{1}{2}$, we may write the following proportion:

$$4:8::9:18,$$

which is read, 4 is to 8 as 9 is to 18.

371. The foregoing proportion may be written in this form:

$$4:8=9:18,$$

or

$$\frac{4}{8} = \frac{9}{18}.$$

372. The four numbers (or quantities) constituting a proportion are classified as :

(a) First and second couplets.

(b) $\left\{ \begin{array}{ll} \text{antecedents} & \text{the first and third terms.} \\ \text{consequents} & \text{the second and fourth terms.} \end{array} \right.$

(c) $\left\{ \begin{array}{ll} \text{extremes} & \text{the first and fourth terms.} \\ \text{means} & \text{the second and third terms.} \end{array} \right.$

373. If three terms of a proportion are given, the remaining term can be ascertained

374. Sight Exercises.

Give the missing number :

1. $\frac{3}{4} = \frac{?}{16}$.

6. $3 \div 4 = ? \div 16$.

2. $\frac{5}{15} = \frac{15}{?}$.

7. $5:15 = 15:?$.

3. $\frac{7}{?} = \frac{21}{24}$.

8. $7:x = 21:24$.

4. $\frac{?}{9} = \frac{20}{36}$.

9. $x:9::20:36$.

5. $\frac{1.5}{3} = \frac{8}{?}$.

10. $1.5:3::8:x$.

375. When four numbers (or quantities) form a proportion, the product of the numbers in the extremes is equal to the product of the numbers in the means.

In the proportion

$$3 \text{ yd.} : 4 \text{ yd.} :: \$12 : \$16$$

we find that

$$3 \times 16 = 4 \times 12.$$

376. *To find the number of units of a missing extreme in a proportion, divide the product of the numbers in the means by the number of units in the given extreme; and to find the*

number of units in a missing mean, divide the product of the numbers in the extremes by the number of units in the given mean.

377. Written Exercises.

Find the missing term :

$$1. \quad 2 \text{ ft. } 6 \text{ in.} : 7 \text{ ft. } 6 \text{ in.} :: 25\phi : x\phi.$$

Reducing the terms of the first couplet to inches, we have

$$30 \text{ in.} : 90 \text{ in.} :: 25\phi : x\phi.$$

The number, x , in the missing term is $\frac{90 \times 25}{30}$;

canceling, we have $x = \frac{90 \times 25}{30} = 75$. *Ans.* 75 ϕ .

$$2. \quad \$6 : \$18 :: x \text{ T.} : 15 \text{ T.}$$

$$3. \quad 400 \text{ cu. ft.} : 1200 \text{ cu. ft.} :: 60 \text{ da.} : x \text{ da.}$$

$$4. \quad 9 \text{ men} : x \text{ men} :: \$306 : \$1020.$$

$$5. \quad ? \text{ days} : 14 \text{ days} :: 108 \text{ rods} : 168 \text{ rods.}$$

$$6. \quad 7 \text{ men} : x \text{ men} :: 22 \text{ A.} : 66 \text{ A.}$$

$$7. \quad \frac{5}{8} \text{ yd.} : 1\frac{1}{4} \text{ yd.} :: x : 9.$$

$$8. \quad 18 \text{ men} : 24 \text{ men} :: 108 \text{ da.} : x \text{ da.}$$

$$9. \quad 48 \text{ hr.} : 45 \text{ hr.} :: \$x : \$15.$$

$$10. \quad 18 \text{ horses} : x \text{ horses} :: \$42 : \$14.$$

378. Oral Problems.

1. If I pay \$18 freight for the transportation of 15 tons, how many tons should be carried for \$6?

2. A certain number of men can build 400 cu. ft. of wall in 60 days. How long would it take them to build 1200 cu. ft.?

3. If $2\frac{1}{2}$ feet of wire cost 25 cents, what should be paid for $7\frac{1}{2}$ feet at the same rate?

4. If 9 yards of cloth cost \$36, how many yards can be bought for \$52?

5. If it requires 14 days for a certain number of men to dig a ditch 168 rods long, how many days will they require to complete 108 yards?

6. Seven men can cut 22 acres of grain in a certain time. How many men would be required to cut 66 acres in the same time?

7. If $1\frac{1}{4}$ yd. ribbon will make 4 ties, how many ties will $\frac{5}{8}$ yard make?

8. Eighteen men receive 108 days of vacation in the summer. How many days will 24 men receive?

9. A man is paid \$15 for 45 hours of work. How much should he be paid for 48 hours of work?

10. If the food of 18 horses costs \$42 for a given period, how many horses can be fed for the same time for \$14?

379. Written Problems.

1. If $\frac{3}{7}$ of a farm is worth \$5130, what is the value of $\frac{5}{9}$ of it?

As the value of a part depends directly on the size of the part, the larger the part the greater the value; the following is the proportion.

Value of $\frac{3}{7}$: value of $\frac{5}{9}$: $\frac{3}{7}$ farm : $\frac{5}{9}$ farm,

or $5130 : x :: \frac{3}{7} : \frac{5}{9}$.

Therefore, $\frac{3}{7}x = 5130 \times \frac{5}{9}$

or, $x = 5130 \times \frac{5}{9} \div \frac{3}{7}$

$= 5130 \times \frac{5}{9} \times \frac{7}{3}$. Cancel.

2. If it takes 810 yards of material 27 inches wide to make a certain quantity of clothing, how many yards of material 30 inches wide would be required to make the same quantity?

In this case the ratio of the number of yards required is in inverse ratio to the width, the greater the width the smaller the number of yards needed.

Quantity of 27 in. : Quantity of 30 in. :: 30 : 27
material : material

810 : x :: $30 : 27$

Canceling, $x = 729$. Ans. 729 yd.

NOTE. For convenience, it is usual to make the missing quantity the fourth term of the proportion.

3. If a certain quantity of food lasts 65 men 12 days, how long should it last 26 men?

$$\begin{array}{ccccccc} \text{men} & & \text{men} & & \text{da.} & & \text{da.} \\ 26 & : & 65 & :: & 12 & : & x \end{array}$$

This arrangement shows more clearly to the beginner that 26 is the divisor, and that the first term can be "canceled" with either of the other two given terms.

4. The number of boys in a certain school is to the number of girls as 8 is to 9. There are 168 boys in the school. How many of the pupils are girls? How many pupils in the school?

5. Two numbers bear to each other the ratio of 5 to 12. The larger number is 204; find the other.

6. At \$4.75 per cord of 128 cu. ft., how much will a pile of wood be worth that measures $16' \times 9' \times 4'$?

7. If a man travels at the rate of 10 miles in 3 hours 20 minutes, how far can he travel in 4 hours 40 minutes?

8. A pole 6 feet high casts a shadow of 4 ft. 8 in. How high is a tree whose shadow at the same time measures 56 feet?

9. If 42 men require 18 days to do a certain piece of work, how many men will be required to do the same work in 27 days?

10. If 49 acres of land produce 2450 bu. corn, how many bushels should be produced by 80 acres?

11. A train travels 224 miles in 5 hours. In what time can it make a trip of 840 miles at the same rate?

12. In raising a stone by means of a crowbar, a pressure of 7 lb. lifts a weight of 49 lb. How much pressure is required to lift a weight of 218 lb.?

13. If 9 barrels of lime are used in laying 8000 common bricks, how many barrels of lime will be required in building a wall containing 120,000 bricks?

14. If the ratio of an avoirdupois pound to a troy pound is 7000 to 5760, how many pounds avoirdupois are equivalent to 175 pounds troy?

15. If the ratio of an avoirdupois ounce to a troy ounce is $437\frac{1}{2}$ to 480, how many ounces avoirdupois are equal to 1000 troy ounces?

16. A cubic foot of water (1728 cu. in.) weighs $62\frac{1}{2}$ lb. Find the weight of 1 gallon of water, 231 cu. in.

17. If a man whose property is valued at \$9000 pays \$82.80 taxes per year, what should be the annual taxes of a man whose property is valued at \$2250?

18. How many tons of 2000 pounds each are equal to 50 tons of 2240 pounds each?

19. If the profits of a business are divided between two partners in the ratio of 12 to 15, and the former receives \$1440 of the profits, how much should the latter receive?

20. A farmer obtains from a field 960 measured bushels of oats weighing 28 pounds to the bushel. How many bushels can he sell at the rate of 32 pounds by weight to the bushel?

PARTITIVE PROPORTION.

380. The process of dividing a given number into parts proportional to given numbers is called *partitive proportion*.

381. Written Problems.

1. A, B, and C receive \$360 for hauling a quantity of wood. A furnishes 2 teams, B 3 teams, and C 4 teams. What is the proportionate share of each?

2 teams

3 teams

4 teams $9:2::\$360:A's\ share.$ $9:3::\$360:B's\ share.$ $9:4::\$360:C's\ share.$

As there were 9 teams at work of which A furnished 2, the whole sum is to A's share as 9:2.

The whole sum is to B's share as 9:3, and to C's share as 9:4.

2. For hauling ice, P, Q, and R receive \$210. P furnishes 2 teams for 8 days, Q 3 teams for 10 days, and R 4 teams for 6 days. How should the money be divided?

$$2 \times 8 = 16$$

$$3 \times 10 = 30$$

$$4 \times 6 = \underline{24}$$

$$70:16::\$210:P's\ share.$$

$$70:30::\$210:Q's\ share.$$

$$70:24::\$210:R's\ share.$$

P furnishes 2 teams for 8 days, which is equivalent to 1 team for 16 days. Q furnishes 3 teams for 10 days, which is equivalent to 1 team for 30 days. R furnishes 4 teams for 6 days, which is equivalent to 1 team for 24 days. The total is the equivalent of 70 days' work, of which P, Q, and R are entitled, respectively, to $\frac{16}{70}$, $\frac{30}{70}$, and $\frac{24}{70}$.

3. Divide 100 into two parts which shall have a ratio to each other of 6 to 9.

4. Divide \$200 between M and N so that M shall receive \$2 for every \$3 received by N.

5. X and Y divide between them \$300, X taking $\frac{9}{11}$ of the sum taken by Y. What does each receive?

6. In mixing concrete for a foundation 2 parts of cement, 3 parts of broken stone, and 4 parts of sand are used. How many cubic yards of each are required to make 2700 cubic yards of concrete?

7. Five farmers pay \$75 for the expense of irrigating their farms, comprising 324 acres, 144 acres, 96 acres, 120 acres, and 216 acres, respectively. What sum should be paid by each?

8. Six families agree to pay a teacher on the basis of the respective number of their children attending the school. How much should each contribute to the \$ 200 paid, if the children number respectively 3, 1, 4, 2, 5, and 1 ?

9. In a school of 357 pupils the number of boys is to the number of girls as 8 to 9. How many boys in the school ? How many girls ?

10. Two numbers whose sum is 714 have the ratio of 5 to 12. What are the numbers ?

PARTNERSHIP.

382. Two or more persons desiring to combine their capital and experience, may form a *partnership* for the purpose of carrying on business of any kind.

383. Each member of a firm taking part in its management is called a *general partner*. In some cases, one or more *special partners* are admitted to a firm.

384. The sharing of the profits or the losses among partners is a matter of agreement. In the problems given under this heading, it is assumed, unless otherwise stated, that profits or losses are shared in proportion to the sums invested.

385. Written Exercises.

1. C, D, and E enter into partnership contributing respectively, \$3500, \$4500, and \$5500. The profits are \$7000. What share of the profits should each receive ?

2. M and N form a partnership, the former contributing \$10,000 and the latter \$5000. It is agreed that N shall receive \$1000 from the profits as compensation for extra experience. What is each one's share of the profits if the year's business shows a profit of \$5500 ?

M gets his share of \$ 4500.

N gets his share of \$ 4500, and \$ 1000 additional.

3. Three men investing \$4500, \$5000, and \$5500 respectively, sell their business at the end of the year for \$11,250. What should each receive as his share?

4. In a mining enterprise four men contribute sums of \$2500, \$3000, \$3500, and \$4000 respectively. They sell the mine for \$12,000 less \$300 commission. What is each one's share of the proceeds?

5. M, N, and O raise cattle as partners, M contributing \$16,000, N \$12,000, and O \$10,000. Their receipts for the year are \$10,000 and their expenses are \$2400. What should each receive from the profits?

POWERS AND ROOTS.

POWERS.

386. By the *square* of 5 is meant the employment of 5 twice as a factor. The square of 5 equals 5×5 , or 25. It may be indicated by placing a small 2 above the 5 to the right, thus, 5^2 .

387. The *cube* of 5 equals $5 \times 5 \times 5$, or 125. It is indicated by writing a small 3 above the 5, to the right, which shows that 5 is to be taken as a factor 3 times. Thus, $5^3 = 125$.

388. The square of 5 is also called the *second power* of 5 and the cube of 5 is called the *third power* of 5.

389. The small number employed to indicate the power is called an *exponent*. To denote that 5 is to be raised to the fourth power; that is, that 5 is to be taken four times as a factor, we use the exponent 4. Thus, $5^4 = 5 \times 5 \times 5 \times 5$; $5^5 = 5 \times 5 \times 5 \times 5 \times 5$.

390. The process of finding the power of a given number is called **involution**.

391. Oral Exercises.

Find:

1. The square of 6.
2. The cube of 4.
3. The fourth power of 3.
4. The fifth power of 2.

Give the values of the following:

- | | | | |
|-----------|-----------|------------|-----------|
| 5. 10^3 | 7. 6^3 | 9. 10^4 | 11. 5^4 |
| 6. 12^2 | 8. 20^2 | 10. 12^3 | 12. 3^3 |

392. Oral Problems.

1. If there are 12 inches in a linear foot, how many square inches are there in a square foot?
2. How many cubic feet in a cubic yard?
3. How many square feet in a square yard?
4. How many cubic inches in a cubic foot?
5. How many square yards in a square rod?

393. Find the square of $20 + 5$.

The product of $(20 + 5)$ by $(20 + 5)$ is equal to the product of 20 times $(20 + 5)$ added to the product of 5 times $(20 + 5)$.

$$\begin{array}{rcl}
 20 \text{ times } (20 + 5) & = & 20^2 + 20 \text{ times } 5 \\
 + 5 \text{ times } (20 + 5) & = & + 20 \text{ times } 5 + 5^2 \\
 \hline
 (20 + 5) \text{ times } (20 + 5) & = & 20^2 + \text{twice } (20 \times 5) + 5^2; \\
 & & \text{or } 400 + 200 + 25, \text{ or } 625.
 \end{array}$$

The square of the sum of two numbers is equal to the square of the first, plus twice the product of the first by the second, plus the square of the second.

394. Oral Exercises.

Give answers:

1. $20^2 + 2 \times (20 \times 2) + 2^2$
2. $(20 + 3) \times (20 + 3)$

3. 31^2

5. 35^2

7. 3.5^2

9. $(3\frac{1}{2})^2$

4. 41^2

6. 45^2

8. 4.5^2

10. $(5\frac{1}{2})^2$

11. A square field measures 65 rods on a side. What is its area in square rods?

12. How many square feet in the ceiling of a square room, one side of which is 42 feet long?

13. The floor of a square room is 14 yards wide. How many square yards does it contain?

14. What is the area of a floor $10\frac{1}{2}$ yd. long and $10\frac{1}{2}$ yd. wide?

15. How many square feet in the six equal square faces of a cube, one edge of which measures 4 inches?

ROOTS

395. To find a **root** of a number means to find one of the equal factors of the number. The *square root* is one of its two equal factors; the *cube root* is one of its three equal factors.

396. The square root of 64 is indicated thus, $\sqrt{64}$. The cube root is indicated thus, $\sqrt[3]{64}$. The 4th root is indicated thus, $\sqrt[4]{16}$.

397. Oral Exercises.

1. $\sqrt{100}$

4. $\sqrt{121}$

7. $\sqrt{25}$

2. $\sqrt{36}$

5. $\sqrt{64}$

8. $\sqrt{81}$

3. $\sqrt{49}$

6. $\sqrt{81}$

9. $\sqrt{144}$

398. Written Exercises.

1. Find the square root of 1296.

Resolving 1296 into its prime factors, we have

$$1296 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3.$$

Separating these factors into two sets containing the same factors, we have $1296 = (2 \times 2 \times 3 \times 3) \times (2 \times 2 \times 3 \times 3)$

Or, $1296 = 36 \times 36$, or $(36)^2$

Therefore, $\sqrt{1296} = 36$. *Ans.*

By the use of factors, find the square root of each of the following:

2. 196

4. 256

6. 225

3. 324

5. 441

7. 484

8. Find the cube root of 1728.

$$1728 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

Separating these factors into three sets containing the same factors, we have $1728 = (2 \times 2 \times 3) \times (2 \times 2 \times 3) \times (2 \times 2 \times 3)$

$$1728 = 12 \times 12 \times 12, \text{ or } (12)^3$$

Therefore, $\sqrt[3]{1728} = 12$. *Ans.*

By the use of factors find the cube root of each of the following:

9. 216

11. 729

13. 3375

10. 576

12. 2744

14. 4096

Extract the roots indicated:

15. $\sqrt[4]{1296}$

16. $\sqrt[5]{1024}$

17. $\sqrt[6]{729}$

SQUARE ROOT.

399. Written Exercises.

1. Extract the square root of 1225.

Since the square of 30 is 900, and the square of 40 is 1600, the square root of 1225 is between 30 and 40; that is, it equals 30 + another number. Calling the missing number N , we have

$$(30 + N) \times (30 + N) = 1225,$$

or

$$30^2 + (2 \times 30 \times N) + N^2 = 1225$$

Deducting 30^2 , or 900, from each side of the equation, we have

$$(60 \times N) + N^2 = 1225 - 900 = 325$$

This may be changed to read as follows:

$$60 \text{ times } N + N \text{ times } N = 325,$$

or

$$(60 + N) \text{ times } N = 325.$$

The problem now becomes

“What number added to 60 and the sum multiplied by the same number equals 325?” Using 60 as a trial divisor, we find that 5 satisfies the conditions; viz., $(60 + 5) \times 5 = 325$.

The process may be shown in this manner

$$\begin{array}{r} 1225(30 + 5 \\ 900 \\ \text{Trial divisor } 60 \quad 325 \\ (60 + 5) \times 5 \quad \underline{325} \end{array}$$

2. Find the square root of 62,001.

In practice, the work is shortened by the omission of ciphers. The number is pointed off in periods of two places each, beginning at the right.

$$\begin{array}{r} 249 \quad \text{Ans. } 249 \\ \underline{6'20'01} \\ 4 \\ 44 \quad \underline{220} \\ 176 \\ 489 \quad \underline{4401} \\ 4401 \end{array}$$

The square root of the greatest square in the first period is 2, which is written above the 6, and its square, 4, is written under the 6. Taking 4 from 6 gives 2 as a remainder, after which are placed the two figures of the second period, making the next dividend 220. The 2 in the root is doubled, giving 4 as a trial divisor, to which is to be annexed the second figure of the root. This is found to be 4, which is written above the next period, and also after the 4 of the trial divisor, which now becomes 44. Multiplying the latter by the 4 in the root gives a product of 176, which is deducted from 220 leaving a remainder of 44. To this are annexed the two figures of the last period, making the next dividend 4401. The 44 in the result is doubled, giving 88 as a trial divisor, to which is to be annexed the third figure of the root. This is found to be 9, which is written above the third period and also after the 88 of the trial divisor, which now becomes 889. Multiplying the latter by the 9 in the root, gives a product of 8001. There being no remainder, 249 is the square root of 62,001.

3. Find the square root of 164,025.

As the first trial divisor, 8 with a figure annexed, is not contained in 40, another period must be brought over, the new trial divisor being 80.

$$\begin{array}{r} 405 \quad \text{Ans. } 405 \\ \underline{16'40'25} \\ 16 \\ 805 \quad \underline{4025} \\ 4025 \end{array}$$

Find the square root of the following :

- | | | |
|--------|--------|-----------|
| 4. 169 | 6. 529 | 8. 123.21 |
| 5. 189 | 7. 616 | 9. 492.84 |

400. In pointing off numbers containing decimals, commence at the decimal point and point off two figures each way, annexing decimal ciphers when necessary.

401. Find the square root of the following (3 figures in the result):

- | | | |
|------|-------------|--------|
| 1. 2 | 3. .3 (.30) | 5. 10. |
| 2. 3 | 4. .5 | 6. 12. |

402. Written Problems.

1. How many rods long is a square field containing 10 acres ?

2. How many yards wide is a square plot of ground containing 1 acre ?

3. A cube has 6 square faces, and its entire surface contains 1350 square inches. What is the volume of the cube?

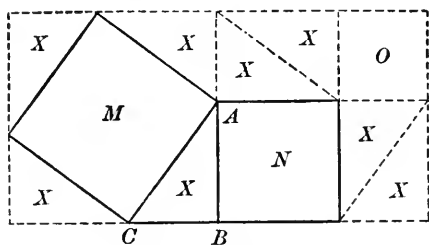
4. What is the side of a square field that will contain the same area as a rectangular field 81 rods wide by 144 rods long ?

5. A man has exchanged two plots, one 240 rods square and the other 320 rods square, for a single square plot equal in area to the combined areas of his two plots. What is the length of a side of the new plot ?

APPLICATIONS OF POWERS AND ROOTS.

403. Draw a right-angled triangle, ABC , having the base BC , the perpendicular AB , and the hypotenuse AC . On the hypotenuse AC , construct a square, and extend BA and BC to form two sides of a square circumscribing the one constructed on AC . Complete this square, then extend the

upper and lower sides to the right and construct a second square equal in area to the other. The first large square



contains the square M and 4 equal right-angled triangles marked X . The other large square contains the squares N and O and 4 equal right-angled triangles marked X . Taking away

the four right-angled triangles from each, we have

$$\text{square } M = \text{square } N + \text{square } O.$$

M is the square constructed on the hypotenuse of the right-angled triangle, N is the square constructed on the perpendicular of the right-angled triangle, and O is a square constructed on the base of a right-angled triangle, of the same dimensions as the triangle ABC .

The square constructed on the hypotenuse of a right-angled triangle is equal to the sum of the squares constructed on the other two sides.

404. Calling the unit of measurement of the hypotenuse H , and the units of the other sides O and B respectively, we have

$$H^2 = P^2 + B^2,$$

or

$$H = \sqrt{P^2 + B^2}.$$

That is, the length of the hypotenuse is equal to the square root of the sum of the squares of the other two sides.

To find the length of the hypotenuse of a right-angled triangle, add together the squares of the lengths of each of the other sides and extract the square root of the sum.

405. Since

$$P^2 + B^2 = H^2,$$

$$B^2 = H^2 - P^2,$$

and

$$P^2 = H^2 - B^2.$$

Therefore,

$$B = \sqrt{H^2 - P^2} \text{ and } P = \sqrt{H^2 - B^2}.$$

To find the length of the base (or the perpendicular) of a right-angled triangle, extract the square root of the difference between the square of the length of the hypotenuse and the square of the length of the other side.

406. Written Exercises.

Find the length of the missing side:

1. $P = 5$ rods, $B = 12$ rods.
2. $H = 13$ yards, $P = 5$ yards.
3. $P = 24$ feet, $B = 7$ feet.
4. $B = 16$ miles, $P = 30$ miles.
5. $H = 45$ inches, $B = 36$ inches.
6. $H = 4\frac{1}{4}$ inches, $P = 3\frac{3}{4}$ inches.
7. $B = 40$ feet, $P = 9$ feet.
8. $H = 61$ yards, $B = 60$ yards.
9. $B = 40$ rods, $P = 40$ rods.
10. $H = 100$ feet, $B = P$.

407. Written Problems.

NOTE. Make diagrams when necessary.

1. A man travels due north 48 miles, then due east 64 miles. How far is he in a straight line from his starting point?
2. Find the distance between the opposite corners of the floor of a room 24 feet long 18 feet wide.
3. A rectangular field is 32 rods long and 60 rods wide. How many acres does it contain? What is the length of the diagonal?
4. A triangle whose base is 60 feet has two equal sides measuring 34 feet each. What is the length of the perpendicular let fall from the apex to the middle of the base?

5. A square field contains 40 acres. What is the length of each side? Find the square of the number that represents the length of the diagonal in rods.

6. If the square of the number of rods in the diagonal of a square field is 1600, what is the square of the number of rods in each side? How many acres in the field?

7. The diagonal of a square field is 25 rods. Find the area of the field in acres?

10 sq. chains = 1 acre.

8. A room is 12 feet long, 9 feet wide, and 8 feet high. How long is the diagonal of the floor? How far is the end of this diagonal from the opposite corner of the ceiling?

9. How far from the foot of a house must a 51-foot ladder be placed so that it will just reach a window 45 feet above the street?

10. A ladder 50 feet long is so placed between two houses that it can just reach the top of each without being moved at the foot. One house is 48 feet high and the other is 40. How far apart are the houses?

CHAPTER VII.

MENSURATION; MISCELLANEOUS PROBLEMS.

AREAS OF PLANE SURFACES.

408. To find the area of a plane surface is to ascertain how many times this surface contains another surface taken as a unit, or how many aliquot parts of this other surface it contains.

409. As a rule, the unit employed as the measure of a surface is a square having for its sides the unit of length. Thus, when the unit of length is the inch, the unit of surface is the *square inch*; when the unit of length is the foot, the unit of surface is the *square foot*; etc.

The *acre* is the only unit of surface that has not a corresponding unit of length.

410. A plane figure bounded by straight lines is called a **polygon**. A polygon of three sides is called a **triangle**; of four sides, a **quadrilateral**; of five sides, a **pentagon**; of six sides, a **hexagon**.

A triangle containing a right angle is called a *right-angled triangle*, or a *right triangle*.

411. Considering the lengths of the sides, a triangle having three equal sides is called an *equilateral* triangle; one having only two equal sides is called an *isosceles* triangle; one in which all the sides are unequal, is called a *scalene* triangle.

412. A **parallelogram** is a quadrilateral whose opposite sides are parallel (Figs. 1-4).

A parallelogram having four square corners is called a *rectangle* (Figs. 1 and 2). When the four sides of a rec-



FIG. 1.



FIG. 2.



FIG. 3.

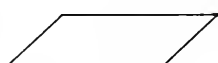


FIG. 4.

tangle are equal, it is called a *square* (Fig. 1); when the adjacent sides are unequal, the term *oblong* is frequently applied (Fig. 2).

413. The *rhombus* (Fig. 3) and the *rhomboid* (Fig. 4) have no square corners; in the former the sides are all equal; in the latter the adjacent sides are unequal.

414. A quadrilateral having only two of its sides parallel



FIG. 5.



FIG. 6.

is called a *trapezoid* (Fig. 5); one having no parallel sides is called a *trapezium* (Fig. 6).

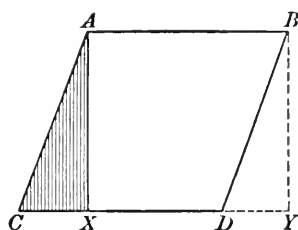
Preliminary Exercises.

415. 1. Cut from a strip of paper 2 inches wide a rhombus $ABDC$ having its sides 3 inches long. The altitude is AX .

A perpendicular that measures the distance between two parallel sides of a quadrilateral is called the *altitude*.

2. Cut from a strip of paper 2 inches wide two rhomboids having two parallel sides each 3 inches long, the remaining two sides of one rhomboid measuring $2\frac{1}{2}$ inches each, and those of the other rhomboid measuring $3\frac{1}{2}$ inches each. What is the altitude of each rhomboid?

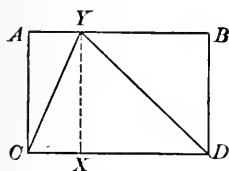
3. From one corner of each of the three parallelograms draw a perpendicular to the opposite side, and cut off a right triangle (ACX). Transfer the triangle to the other side (BDY). What are the dimensions in each case of the rectangle thus formed?



The number of square units in the area of a parallelogram is equal to the product of the number of units in the base by the number in the altitude.

NOTE. The dimensions must be expressed in the same units before performing the multiplication.

4. From a paper rectangle 3 inches by 2 inches, $ABDC$, cut a triangle YCD , Y being taken at any point on AB .



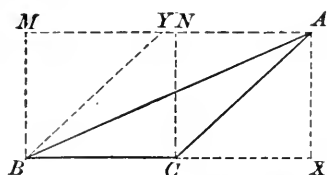
Place the remaining triangles on the triangle YCD to show that together they are equal to YCD and that the triangle YCD , therefore, is one-half of the rectangle $ABDC$. How many

square inches are there in the area of the rectangle? What is the area of YCD ?

The number of square units in the area of a triangle is equal to one-half the product of the number of units in the base by the number in the altitude.

The line that measures the altitude of a triangle may lie outside of the triangle.

In the accompanying figure, considering BC as the base of the triangle ABC , the altitude is AX . Its length, however, is the same as that of MB or NC .



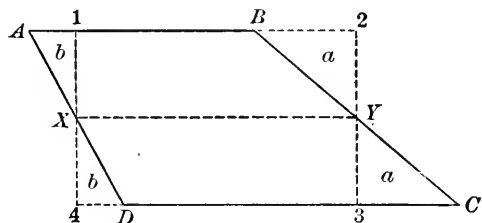
The triangle $ABC = \frac{1}{2}$ parallelogram $AYBC = \frac{1}{2}$ rectangle $NMBC$.

$$\text{Area} = \frac{1}{2} (BC \times AX).$$

Any side of a triangle or of a parallelogram may be taken as the base, the altitude in each case being the perpendicular let fall on the base (or the base produced) from the vertex of the opposite angle.

NOTE. The *dimensions* of a triangle or of a parallelogram are the base and the altitude, as these determine the area, whatever may be the shape of the figure.

416. Cut from a strip of paper 2 inches wide several trapezoids, making one parallel side $2\frac{1}{2}$ inches long in each case and the other $3\frac{1}{2}$ inches long.



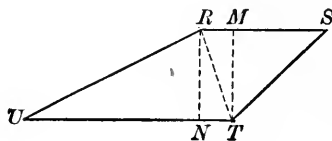
Fold AB over on DC , creasing the paper along XY . Measure XY . Cut off the triangle $Y3C$ and place it at $B2Y$.

Cut off $A1X$ and place it at $X4D$.

What are the dimensions of the rectangle thus formed?

A trapezoid is equal in area to a rectangle whose dimensions are the altitude of the trapezoid and the half sum of the parallel sides.

The trapezoid $RSTU$ is divided into two triangles by RT .



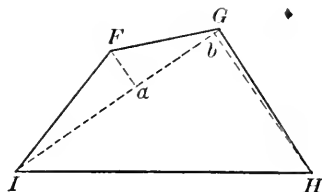
$$\text{Area of } RTS = \frac{1}{2} RS \times MT$$

$$\text{Area of } RUT = \frac{1}{2} UT \times MT(RN)$$

Adding,
$$\text{Area of } RSTU = \frac{1}{2} (RS + UT) \times MT$$

The number of square units in the area of a trapezoid is equal to the product of one half the sum of the units in the parallel sides by the number in the altitude.

417. In the trapezium $FGHI$, a diagonal GI is drawn, and perpendiculars Fa and Hb are let fall upon it from the vertex of each of the opposite angles. The trapezoid is thus divided into two triangles whose areas are as follows:



$$\begin{array}{l} \text{Area of } FGI = IG \times \frac{1}{2} Fa \\ \text{Area of } GHI = IG \times \frac{1}{2} Hb \\ \hline \text{Area of } FGHI = IG \times \frac{1}{2} (Fa + Hb) \end{array}$$

The number of square units in the area of a trapezoid is equal to the product of the number of units in the diagonal by one half the sum of the units in the two altitudes let fall on the diagonal.

AREA OF A POLYGON.

418. Written Problems.

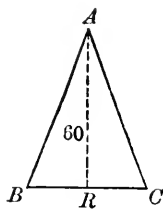
NOTE. In some of the following problems the length of a side is to be obtained from a given area or other data. In others, a missing dimension must be calculated before a required area can be obtained. The employment of a diagram on which are noted the items given will frequently aid the pupil to determine the steps necessary to be taken. The only difficulty in these problems consists in determining the operations required.

1. Two rectangular fields are equal in area; one measures 73.10 rods by 28.80 rods; the length of the second is 59.80 rods. What is the width of the second field? (Cancel.)

2. How many acres in a rectangular field having one side 75 rods long, and its diagonal measuring 125 rods?

3. A classroom 30 feet long has a floor space of 600 square feet. It is desired to increase the width of the room to accommodate 40 boys, giving each 2 square yards of floor

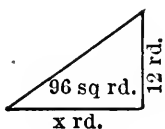
space. How many feet must be added to the width of the room?



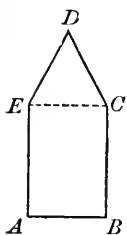
4. A 12-acre field has the form of an isosceles triangle whose altitude measures 60 rods. Find the sides of the triangle.

$$\frac{60BC}{2} = 1920. \quad BC = ? \quad CR = ? \quad AC = \sqrt{60^2 + CR^2}.$$

5. How many yards of fencing will be required to inclose a plot of ground in the form of a right triangle having a perpendicular of 12 rods and containing $\frac{3}{5}$ acre?

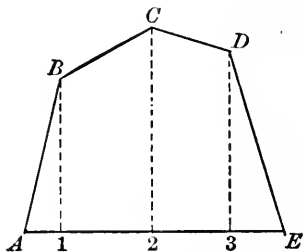


$$\frac{2 \times \text{Base}}{2} = 96. \quad \text{Find hypotenuse.}$$

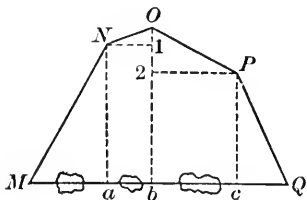


6. How many square yards are there in the gable end of a house, the measurements being: AB , 30 feet; AE , 36 feet; DE and DC , each 25 feet?

7. A pentagonal field has a base AE , 100 rods long. Perpendiculars to AE from the other three corners measure as follows: $B1$, 60 rods; $C2$, 80 rods; $D3$, 70 rods. The points 1, 2, and 3 are, respectively, 15, 50, and 80 rods distant from A . How many square rods does the field contain?



8. Owing to the swampy ground it is difficult to ascertain the length of MQ by actual measurement. The other sides have lengths as follows: MN , 17 rods; NO , 5 rods; OP , 10 rods; PQ , 13 rods. The following are the lengths of the perpendiculars to MQ : Na ,



15 rods; *Ob*, 18 rods; *Pc*, 12 rods. Find the area of the field, and the length of *MQ*.

Ma is the base of a right-angled triangle, the perpendicular being *Ma*, and *MN* the hypotenuse. Using *ON* and *OI*, the length of *NI* (or its equal *ab*) can be ascertained; etc.

9. Find the altitude of an equilateral triangle having sides of 6 feet each. Find its area.

10. Find the altitude and the area of an equilateral triangle each of whose sides measures 1 foot.

To find the Area of a Triangle when the Lengths of the Sides are Given.

419. In many cases a person desirous of ascertaining the contents of a triangular piece of ground finds it difficult to locate and measure the altitude. In such a case the area is obtained from the lengths of the sides by the following formula: Calling half of the perimeter *h*, and the sides *a*, *b*, and *c*, respectively,

$$\text{Area} = \sqrt{h \times (h - a) \times (h - b) \times (h - c)}.$$

420. *The number of square units in the area of a triangle is equal to the number of units in the square root of the combined product of the units in one half of the perimeter by the respective differences between the units in the half perimeter and the units in each side, successively.*

421. The following examples will illustrate the method:

1. Find the area of a triangle whose sides measure 16, 20, and 28 yards, respectively.

$$h = \frac{1}{2}(16 + 20 + 28) = 32$$

$$h - a = 32 - 16 = 16$$

$$h - b = 32 - 20 = 12$$

$$h - c = 32 - 28 = 4$$

$$\text{Area} = \sqrt{32 \times 16 \times 12 \times 4} = \sqrt{24756} = 156.76$$

Ans. 156.76 sq. yd.

2. The sides of a triangular field measure 100 rods, 156 rods, and 224 rods, respectively. How many acres does the field contain?

$$\frac{1}{2} \text{ of } (100 + 156 + 224) = 240$$

$$240 - 100 = 140$$

$$240 - 156 = 84$$

$$240 - 224 = 16$$

$$\text{Area in square rods} = \sqrt{240 \times 140 \times 84 \times 16} = 6720$$

$$\text{Area, 6720 sq. rd.} = 42 \text{ acres.}$$

$$\text{Ans. 42 acres.}$$

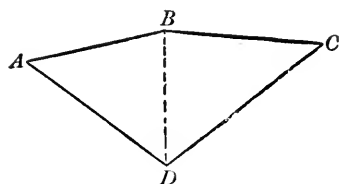
3. The boundaries of a triangular farm measure 340 rods, 200 rods, and 420 rods, respectively. How many acres are there in the farm?

4. Find the number of square rods in a triangular piece of woodland whose sides are, respectively, 52 rods, 56 rods, and 60 rods.

5. How many acres are contained in a triangular park whose sides are 52 rods, 148 rods, and 160 rods, respectively?

6. A piece of ground in the form of a triangle has sides of 45 feet, 111 feet, and 132 feet, respectively. How many square yards does it contain?

7. Find the area in square rods of a quadrilateral measuring as follows:



AB , 14 rods; BC , 20 rods; CD , 21 rods; and DA , 15 rods. The length of the diagonal BD is 13 rods.

NOTE. Find the area of each triangle separately. Make a diagram and write on each side its length.

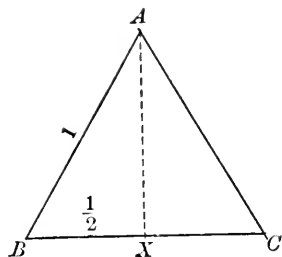
AREA OF A REGULAR HEXAGON.

422. Exercises.

1. Find the altitude of an equilateral triangle whose sides are 1 inch. (See cut on next page.)

$$AX = \sqrt{AB^2 - BX^2} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2} = \frac{1}{2}\sqrt{3}$$

2. What is the area of an equilateral triangle whose sides are each 4 inches?



The area may be obtained by finding the altitude and multiplying this by one half the base, or by the method given in Art. 420.

$$\text{Area in square inches} = \sqrt{6 \times 2 \times 2 \times 2}.$$

3. Find the area of a hexagon composed of six equal equilateral triangles, each side of a triangle measuring 2 inches.

The apothem of a regular polygon is the line drawn from the center of the polygon to the middle point of one side.

4. Find the apothem of a regular hexagon whose side is 1 inch. (See No. 1.)

5. Find the apothem of a regular hexagon whose side is 2 inches. Whose side is 4 inches.

$$\text{The area of a regular polygon} = \frac{1}{2} (\text{perimeter} \times \text{apothem}).$$

6. How many square feet of surface will be covered by 1000 hexagonal tiles, each 6 inches on a side?

UNITED STATES PUBLIC LANDS.

423. Land owned by the United States government is laid out in *townships* six miles square, which are divided into *sections* one mile square, containing 640 acres.

424. Townships are located with reference to a line called the *principal meridian* and a *base line* intersecting the former at right angles. The townships are bounded by lines running north and south, 6 miles apart, parallel to the principal meridian, and by east and west lines, also six miles apart, parallel to the base line.

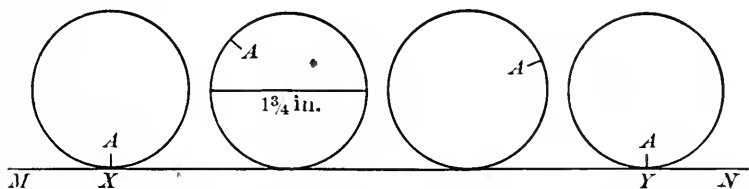
West half, according to its location. The North half in Fig. 3 is divided into two quarter sections, as is the West half.

428. Sight Exercises.

1. What is the value of S.E. $\frac{1}{4}$ of Sec. 27, T. 7 N., R. 15 W., at \$20 per acre?
2. How many square rods in S.E. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ of Sec. 25, T. 5 S., R. 15 W.?
3. How many rods of fence will be required to inclose S.W. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ of the same section?
4. A farmer plants the W. $\frac{1}{4}$ of S.W. $\frac{1}{4}$ of the same section in corn. How many bushels of corn does he obtain to the acre, if the total yield is 4000 bushels?
5. If the owner sold the West half of the North half of the same section, how would the part sold be designated? What would be the designation of the part retained?

CIRCUMFERENCE AND AREA OF A CIRCLE.

429. Carefully cut out of cardboard a circle having a diameter of $1\frac{3}{4}$ inches. Make a dot A on the circumference, and placing the dot at the point X on a sheet of paper, revolve the circle until the dot again touches the paper at Y , which is marked. Measure the distance XY , which should be $5\frac{1}{2}$ inches.



430. The ratio between the diameter and the circumference is the ratio between $1\frac{3}{4}$ inches and $5\frac{1}{2}$ inches, or

7 to 22. The circumference is, therefore, $3\frac{1}{7}$ times the diameter.

431. *To determine the length of the circumference of a circle multiply the length of the diameter by $3\frac{1}{7}$.*

NOTE. Circumference = diameter $\times 3.1416$ is more accurate when the circle is very large. Unless otherwise specified, use $3\frac{1}{7}$.

432. Sight Exercises.

Find the circumferences of circles having diameters as follows:

- | | | |
|-----------|-----------|-----------|
| 1. 14 yd. | 4. 35 rd. | 7. 63 in. |
| 2. 28 ft. | 5. 49 yd. | 8. 42 mi. |
| 3. 21 in. | 6. 70 rd. | 9. 56 ft. |

Find the diameters of circles having circumferences as follows:

- | | | |
|-------------|-------------|-------------|
| 10. 220 in. | 13. 66 yd. | 16. 110 mi. |
| 11. 44 rd. | 14. 198 ft. | 17. 88 ft. |
| 12. 132 rd. | 15. 176 rd. | 18. 154 in. |

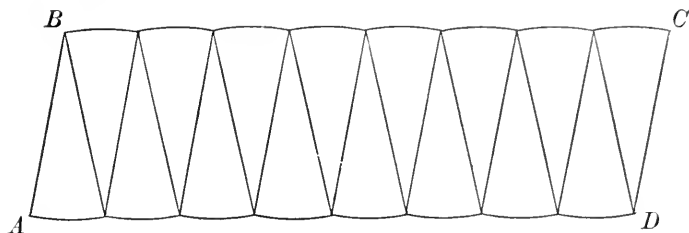
Find the radii of circles having circumferences as follows:

- | | | |
|-------------|-------------|-------------|
| 19. 132 rd. | 22. 44 rd. | 25. 440 in. |
| 20. 88 yd. | 23. 110 ft. | 26. 220 rd. |
| 21. 176 mi. | 24. 264 in. | 27. 308 ft. |

Find the circumferences of circles having radii as follows:

- | | | |
|------------------------|-------------------------|-------------------------|
| 28. $3\frac{1}{2}$ in. | 31. 7 ft. | 34. 28 mi. |
| 29. 35 ft. | 32. $24\frac{1}{2}$ yd. | 35. $10\frac{1}{2}$ rd. |
| 30. 14 rd. | 33. 21 rd. | 36. $31\frac{1}{2}$ yd. |

433. Draw on stiff paper a circle having a radius of $3\frac{1}{2}$ inches. Divide it into 16 equal parts by cuts passing through the center. Place these in two rows, as shown in the accompanying diagram.



434. By increasing the number of pieces, AD gradually approaches a straight line 11 inches long, one half the length of the circumference; and AC gradually approaches a perpendicular measuring $3\frac{1}{2}$ inches, one half the length of the diameter.

435. When the number of pieces becomes indefinitely large, the figure becomes a rectangle 11 inches by $3\frac{1}{2}$ inches.

436. Substituting π for $3\frac{1}{7}$, the ratio between the circumference of a circle and its diameter, and R for the radius of the circle, the line AD , which equals one half of the circumference, will be πR , and AB will be R .

The area of a circle, therefore, is πR^2 ; that is, the square of the radius multiplied by $3\frac{1}{7}$.

NOTE. The Greek letter π (pronounced pī) is used in formulas to indicate the ratio of the length of the diameter to that of the circumference. It stands for $3\frac{1}{7}$, or, more accurately, 3.1416.

437. *The number of square units in the area of a circle is equal to the square of the number of units in its radius multiplied by $3\frac{1}{7}$.*

438. Sight Exercises.

Find the areas of circles with dimensions as given on the following page :

$$\text{Area} = \frac{1}{2} \text{ circumference} \times \frac{1}{2} \text{ diameter.} \quad \text{Area} = 3\frac{1}{7} R^2.$$

Use either formula.

- | | |
|------------------------------|--------------------------------------|
| 1. Radius $3\frac{1}{2}$ ft. | 6. Circumference $6\frac{2}{7}$ ft. |
| 2. Diameter 7 yd. | 7. Radius 2 yd. |
| 3. Circumference 22 in. | 8. Diameter 6 rd. |
| 4. Radius 1 rd. | 9. Circumference $18\frac{6}{7}$ rd. |
| 5. Diameter 2 mi. | 10. Radius 7 in. |

439. Written Problems.

1. If the diameter of a wheel of a wagon is 4 feet, how far does the wagon go during one revolution of the wheel? How many times will the wheel revolve in going 5280 feet?

2. If a wagon wheel 4 feet in diameter revolves 420 times in an hour, how long will it take to travel 1 mile?

3. How many revolutions per minute are made by a wagon wheel 4 feet in diameter when the wagon travels at the rate of 3 miles per hour?

4. Find the number of revolutions per minute made by a locomotive wheel 4 feet in diameter when the train is traveling at the rate of a mile in 2 minutes.

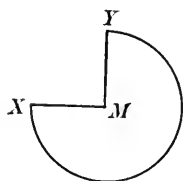
5. How many feet are there in an arc of 60° of a circle whose radius measures 6 feet?

6. Find the radius of a circle whose area is 154 square inches. Find the diameter. Find the circumference.

$$3\frac{1}{2} R^2 = 154; R^2 = 154 \div 3\frac{1}{2}$$

$$R = \sqrt{154 \div 3\frac{1}{2}}$$

7. What is the length of the circumference of a circle containing an area of 616 square inches?

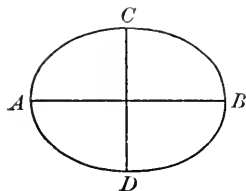


8. A horse is tied at M , the corner of a fence, by a rope 20 feet long. How many square feet of surface can he graze over if he can reach one foot beyond the end of the rope?

$$MX = MY = 21 \text{ feet.}$$

9. An elliptical flower bed measures 14 feet on the axis AB by $10\frac{1}{2}$ feet on axis CD . How many square feet does it contain?

$$\text{Area} = \pi \times \frac{AB}{2} \times \frac{CD}{2}.$$



10. The long axis of an elliptical pond measures 21 yards, and the short axis 9 yards 1 foot. How many square yards are there in its surface?

PRISMS.

440. A **solid** is a portion of space bounded by surfaces. When the side surfaces are parallelograms, and the ends or bases are equal parallel polygons, the solid is called a **prism**.

A prism may be *triangular*, *rectangular*, *pentagonal*, etc., according to the number of sides in its bases.

441. A **right prism** is one whose sides are rectangles.

Fig. 1 shows a right, square prism.

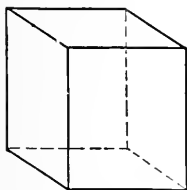


FIG. 1.

442. The *altitude* of a prism is the perpendicular distance between the planes of its bases. AX , Fig. 2, is the altitude of the oblique triangular prism.

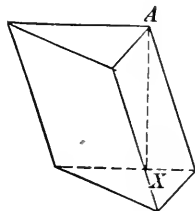


FIG. 2.

NOTE. Unless otherwise specified, the term *prism* is understood to mean a *right prism*.

443. The *convex surface* of a prism is the surface exclusive of its bases; the *entire surface* includes the surface of both bases.

444. In Fig. 3 (page 222) is represented a hollow cardboard prism opened out to show its surface. This is called the *development* of a solid. AD shows the height of the prism, ab one side of the base and ad the other.

445. Sight Exercises.

Assuming the prism, whose development is shown in Fig. 3, to be 8 inches high, and the base to be 4 inches by 2 inches:

1. Find the area of $Bdfc$.
2. Find the area of $adfe$.
3. Find the area of $abcd$.
4. Find the area of $ABCD$.
5. Find the area of rectangle 1.
6. Find the area of rectangle 2.
7. Find the convex surface.
8. Find the entire surface.

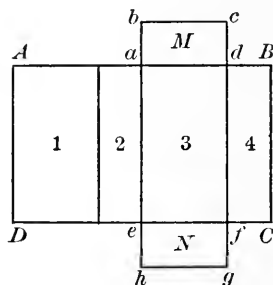


FIG. 3.

9. What is the length of the base of the rectangle forming the convex surface?
10. What is the length of the perpendicular of the rectangle forming the convex surface?

446. *The number of square units in the convex surface of a right prism is equal to the product of the number of units in the perimeter of the base by the number of units in the height.*

The entire surface = convex surface + area of bases.

447. Written Exercises.

1. How many square yards in the convex surface of a hexagonal prism, each side of the base measuring 3 feet, and the height being 10 feet?
2. The base of a prism is a triangle whose sides measure 3 inches, 4 inches, and 5 inches, respectively. Its altitude is 12 inches. How many square inches in the entire surface?
3. The entire surface of a square prism contains 112 square feet. Each side of the base measures 4 feet. What is the convex surface of the prism? Find its altitude.

4. What is the edge of a cube whose entire surface contains 486 square inches?

5. A prism 6 feet high has a rectangular base 2 feet 8 inches by 1 foot 6 inches. How many square feet in the entire surface? Would the convex surface of the prism be greater or less if the base were a square containing the same area as the rectangle?

CYLINDERS.

448. A cylinder, Fig. 4, is a solid having the same diameter throughout and two equal and parallel circles for bases.

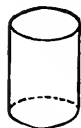


FIG. 4.

Cylinders, like prisms, are either *right* or *oblique*.

The developed surface of the cylinder is shown in Fig. 5.

The line AB is equal in length to the length of the circumference of the circle forming the base.

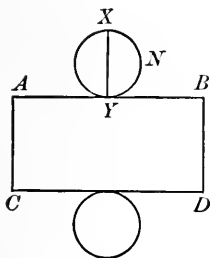


FIG. 5.

449. Sight Exercises.

Assuming the cylinder, whose development is shown in Fig. 5, to be 10 inches high, and the diameter of the base 7 inches, find :

1. The length of AB .
2. The area of $ABDC$.
3. The area of the upper base.
4. The area of the lower base.
5. The convex surface.
6. The entire surface.

450. Written Exercises.

1. Find the convex surface of a cylinder 14 feet high, the diameter of the base being 3 feet. Find the entire surface.
2. At 36 cents a square yard, find the cost of polishing the convex surface of a cylindrical marble column 12 feet high

and 11 feet in circumference. What will it cost to smooth off the ends at 9 cents a square yard?

3. A roller is 6 feet long and $3\frac{1}{2}$ feet high. How many square feet of ground does it roll at each revolution? How many revolutions does it make in going the length of a square field containing 10 acres? How many times must it cross the field to roll the whole field? How many miles must it travel, exclusive of turns?

4. The convex surface of a cylinder 6 feet high is 132 square feet. Find the circumference of the base. Find the radius of the base. Find the area of each base. Find the entire surface of the cylinder.

5. The entire surface of a cylinder is $113\frac{1}{4}$ square inches, the base being 4 inches in diameter. What is the convex surface of the cylinder? What is its altitude?

VOLUME OF PRISM AND CYLINDER.

451. *The number of cubic units in the volume of a prism or of a cylinder is equal to the product of the number of square units in the base by the number of linear units in the altitude.*

452. Written Exercises.

1. What is the volume of a triangular prism whose altitude is 9 feet, the sides of the base measuring 3, 4, and 5 feet, respectively?

2. A cylinder is 12 feet high and 11 feet in circumference. How many cubic feet does it contain?

3. Find the dimensions and the cubical contents of the largest square prism that can be made from a cylindrical wooden column 12 feet high and 11 feet in circumference. How many cubic feet of wood will be removed in making the change?

4. A grindstone $3\frac{1}{2}$ feet in diameter and 4 inches thick has an opening through the center 4 inches square. How many

pounds does it weigh at 135 pounds to the cubic foot?

5. How many square inches are there in the entire surface of a cylinder 7 inches in diameter and 7 inches high? How many cubic inches in its volume?

PYRAMIDS AND CONES.

453. A **pyramid** is a solid whose base is a polygon, and whose sides are triangles terminating in a common vertex.

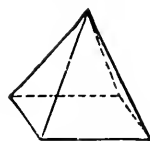


FIG. 6.

454. A **cone** is a solid sloping regularly to its vertex from a circular base.

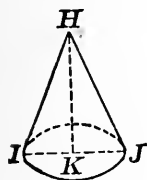


FIG. 7.

455. In a *right* cone or a *right* pyramid the vertex is directly over the center of the base, the sides of a right pyramid being isosceles triangles. In a *regular right* pyramid these triangles are equal.

456. In Fig. 8 is shown the development of a square pyramid, AX denoting the altitude of each isosceles triangle. This is called the *slant height* of the pyramid.

457. Sight Exercises.

Assuming that the slant height AX of the pyramid in Fig. 8 is 12 inches and that DG is 10 inches:

1. Find the area of $DEFG$.
2. Find the area of ADG .
3. Find the area of AGH .
4. Find the area of ADC .
5. Find the area of ABC .
6. What is the convex surface of the pyramid?

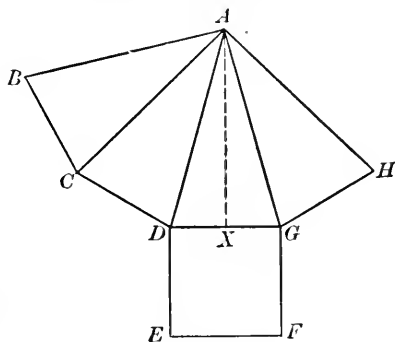


FIG. 8.

7. What is the entire surface ?
8. The convex surface of the pyramid is equal to the slant height multiplied by what ?
9. What is the perimeter of the base $DEFG$?
10. If AX is 12 inches and XD is 5 inches, what is the length of the hypotenuse AD ?

458. *The number of square inches in the convex surface of a pyramid (or of a cone) is equal to one half the product of the number of inches in the perimeter (or the circumference) by the number in the slant height.*

459. Written Exercises.

1. Find the number of square feet in the convex surface of a hexagonal pyramid, each side of the base measuring 1 foot 6 inches, and the slant height being 4 feet 4 inches.
2. What is the entire surface of a square pyramid whose slant height is 3 yards 1 foot, each side of the base being 27 inches ?
3. What is the convex surface of a cone whose slant height is 14 inches, the circumference of the base being 22 inches ?

NOTE. The development of the convex surface of a cone is a sector, the number of square units in the area being equal to the product of the number of linear units in the arc by the number in the radius.

4. The altitude HK of a cone, Fig. 7, is 24 inches and the diameter of the base is 14 inches. Find the slant height HI . Find the convex surface of the cone.

$$HI = \sqrt{HK^2 + KI^2} = \sqrt{24^2 + 7^2}$$

5. The steeple of a church is in the form of an octagonal pyramid, each side of the base measuring 6 feet. How many square yards are there in its convex surface, if the slant height is 42 feet ?

VOLUME OF PYRAMID AND CONE.

460. From a semicircular piece of stiff paper, Fig. 9, make a hollow cone, Fig. 10. With a rectangular piece of stiff paper, Fig. 11, make a hollow cylinder, Fig. 12; the width of the paper, MN , being exactly the altitude of the cone, AB .

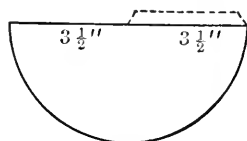


FIG. 9.

Use the cone to fill the cylinder with sand, holding the bottom of the cylinder against a plate to prevent the escape of the sand. How many cones of sand will the cylinder contain?

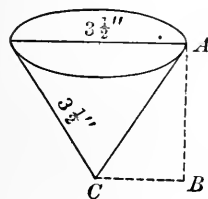


FIG. 10.

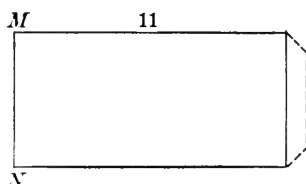


FIG. 11.

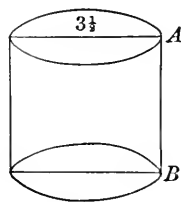


FIG. 12.

461. *The number of cubic units in the volume of a pyramid or of a cone is equal to one-third of the product of the number of square units in the base by the number of linear units in the altitude.*

462. Written Exercises.

1. Find the volume of a square pyramid 12 feet high, each side of the base measuring $5\frac{1}{2}$ feet.

2. A cylindrical can is 7 inches in diameter and 7 inches high. A cone of the same dimensions is placed in the can. How many cubic inches of water will be required to fill the remaining space?

3. A pyramid 12 inches high has a base 10 inches square. If a pyramid 6 inches high is cut from the top by a plane parallel to the base, what will be the dimensions of the base of the small pyramid. How does the volume of the small pyramid compare with the volume of the original pyramid?

4. At $1\frac{1}{4}$ cubic feet to the bushel, find the number of bushels that will be contained in a conical pile of wheat 22 feet in circumference at the base and 7 feet high.

5. Find the number of bushels in a pile of grain in the corner of a granary, if the pile is 7 feet high, each point of the base being $3\frac{1}{2}$ feet from the corner.

What part of a cone is formed by the pile?

THE SPHERE.

463. A **sphere** is a solid, all points on the surface of which are equally distant from its center.

464. The sphere, the right cylinder, and the right cone are called *solids of revolution*.

465. The *cone* is formed by the revolution of the right triangle ABC , Fig. 13, on the perpendicular AB as its axis, the point C of the base BC of the triangle describing the circumference $DNCM$, which bounds the base of the cone.

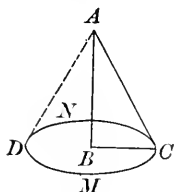


FIG. 13.

466. In the same way, the *cylinder*, Fig. 14, is formed by the revolution of the rectangle $STQR$, on the axis SR , the side RQ forming the base, and TQ the curved surface.

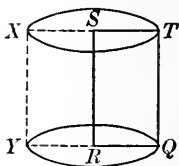


FIG. 14.

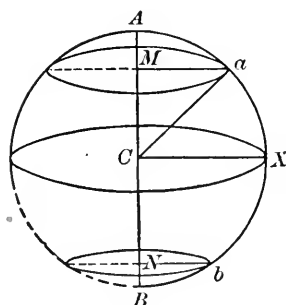


FIG. 15.

467. The sphere is formed by the revolution of a semicircle, AXB , Fig. 15, on the diameter AB as the axis. The radius CX , perpendicular to the axis, describes a *great circle* of the sphere. The half chords Ma and Nb , perpendicular to the axis, describe *small circles*.

468. A cutting plane passed through a sphere in any direction will leave the cut surface a *circle*. If the plane passes through the center of the sphere, the cut surface will be a great circle.

469. The radius of a sphere is a line from the center of a sphere to its surface. CA , Ca , CX , and CB , Fig. 15, are radii.

470. Drive a tack into the center of the curved surface of the half of a croquet ball or other wooden hemisphere. Starting at the tack, wind a stout cord about the curved surface in the way a top is wound. When this surface is entirely covered, cut the cord. Drive another tack into the center of the plane surface, and carefully wind the same cord on the plane surface. When this task is finished, it will be found that just one half of the piece of cord is required. The area of the curved surface is, therefore, twice the area of the plane surface.

471. The area of the plane surface of the hemisphere is the area of a great circle of the sphere. The area of the convex surface of the hemisphere is, therefore, equal to the area of two great circles; and the area of the entire surface of the sphere is equal to the area of four great circles.

472. This may be expressed as follows :

$$\text{Surface of sphere} = 4 \pi R^2;$$

and, since the square of the diameter is 4 times the square of the radius, the formula may be expressed thus :

$$\text{Surface of sphere} = \pi D^2.$$

473. *The number of square units in the surface of a sphere is equal to the square of the number of units in the diameter multiplied by $3\frac{1}{4}$.*

VOLUME OF A SPHERE.

474. Mold carefully a lump of clay into a good-sized ball. Make a stout paper cylinder having its altitude and the diameter of its base equal to the diameter of the ball. Make a paper cone of the same base and altitude as the cylinder. Place the ball in the cylinder and pour into the cylinder the cone full of water, which exactly fills it.

475. The volume of the sphere is equal to the volume of the cylinder less the volume of the cone.

Volume of cylinder = circle \times altitude.

Volume of cone = circle $\times \frac{1}{3}$ altitude.

Volume of sphere = circle $\times \frac{2}{3}$ altitude.

The area of the circle being πR^2 , the volume of the sphere = $\pi R^2 \times \frac{2}{3} D$ or $\pi R^2 \times \frac{4}{3} R$, which equals $\frac{4}{3} \pi R^3$. Substituting $\frac{1}{8} D^3$ for R^3 , we have, area = $\frac{4}{3} \pi \times \frac{1}{8} D^3$, or $\frac{1}{6} \pi D^3$.

476. *The number of cubic units in the volume of a sphere is equal to one sixth of the product of the number of units in the cube of the diameter by $3\frac{1}{2}$.*

477. NOTE. If a sphere, a cylinder, and a cone of the same material are available, the weight of the cylinder should equal the weight of the other two, provided the altitude of the cylinder and of the cone, and the diameter of the base of each, are equal to the diameter of the sphere. If the water that fills a rubber ball is squeezed into a cone having the same diameter as the inner diameter of the ball and having its altitude equal to the diameter, it should fill the cone twice.

478. Written Exercises.

1. Find the surface of a sphere whose diameter is 7 inches.

2. What is the entire surface of a cylinder 7 inches high and 7 inches in diameter?

3. Find the volume of a sphere whose diameter is 7 inches.

4. Find the volume of a cone 7 inches high and 7 inches in diameter at the base. Find the volume of a cylinder 7 inches high and 7 inches in diameter. Find the difference between the two volumes.

5. If a sphere 7 inches in diameter is placed in a hollow 7-inch cube, how many cubic inches of water will the cube then hold?

6. A hemispherical bowl of iron is 7 inches in diameter. How many cubic inches of iron does it contain if the iron is 1 inch thick?

Find the difference between the volume of a 7-inch hemisphere and that of a 6-inch hemisphere.

7. Find the cost of gilding a ball 14 inches in diameter at 36 cents per square foot.

8. If cast iron weighs 7 times as much as water, find the weight of a solid iron ball 6 inches in diameter, the weight of a cubic foot of water being 1000 ounces.

9. What is the ratio between the volume of a 3-inch sphere and that of a 6-inch sphere?

10. What is the largest sphere that can be cut from a cubical block of granite 12 inches on each edge? What decimal of a cubic foot of material will be cut away?

MISCELLANEOUS PROBLEMS.

479. Miscellaneous Oral Problems.

1. How many rods of fence will be required to inclose a square 10-acre field?

2. A contractor has 30 days in which to finish a piece of work. His force of 60 men would require 50 days to complete it. How many additional men must he employ?

3. What are the cubical contents of a cellar 20 feet long, 15 feet wide, and 10 feet deep?

4. If 5 sheep are worth \$ 20, how many calves at \$ 3 each must be given in exchange for 12 sheep?

5. Three fifths of a yard of ribbon costs $7\frac{1}{2}$ cents. How many yards can be bought for \$ 5?

6. In dividing 300 firecrackers among some boys there are 6 left after each boy receives 14. How many boys are there?

7. Find the cost of 16 bushels of wheat at $99\frac{1}{2}$ cents per bushel.

8. Eight men cut 20 cords of wood per day. How long will it take 6 men to cut 80 cords?

9. The contents of a rectangular box are 136 cubic feet. Its length is 6 feet and its depth is 4 feet. Find its width in feet and inches.

10. William and Thomas can do in 8 days a piece of work which the latter can do alone in 12 days. How long would it take William to do it alone?

11. How many rectangular blocks each 2 inches by 3 inches by 4 inches will be required to fill a box measuring 2 feet by 3 feet by 4 feet?

12. When eggs are sold at 15 cents per dozen, a profit of $\frac{1}{4}$ the cost is made. What did they cost?

13. Find the solid contents of a cube, the area of each face of which is 400 square inches.

14. What will be the value of a pile of wood 4 feet wide, 12 feet long, and 4 feet high, at \$4.50 per cord?

15. Find the cost of 49 pounds of tea at 41 cents per pound.

16. Each face of a cube contains 81 square inches. Find its volume.

17. What is the hypotenuse of a right triangle if the other sides measure 6 and 8 feet, respectively?

18. At 18 cents per square yard, what will be the cost of a blackboard 10 feet long, 4 feet wide?

19. How many acres in a field 40 rods long, 40 rods wide?

20. What is the side of a square field containing 64 times as many acres as a square field 40 rods on a side?

21. How many 3-inch cubes can be placed in a box 12 inches long, 9 inches wide, 6 inches high? How many 1-inch cubes can be placed in the same box?

22. If $\frac{4}{5}$ bushel occupies a cubic foot, how many bushels will there be in a wagon body that holds just a cubic yard?

23. How many square rods in a field 65 rods long, 65 rods wide?

24. Find the number of board feet in a stick of timber 12 feet long, 8 inches wide, 3 inches thick.

25. If there are 27 bricks in a cubic foot, and $22\frac{1}{2}$ bricks with the mortar make a cubic foot of wall, what part of the wall is mortar?

26. Making no allowance for openings, how many rolls of paper will be needed for the walls of a room 20 feet long, 15 feet wide, and 10 feet high, 3 rolls being required for 100 square feet?

27. At $1\frac{1}{4}$ cubic feet to the bushel, how many bushels can be placed in a bin 10 feet long, 8 feet wide, and 4 feet deep?

28. How many pounds of flour are there in 25 barrels each containing 196 pounds?

29. How many bricks are there in a cubic foot, if a brick measures $\frac{2}{3}$ foot by $\frac{1}{3}$ foot by $\frac{1}{6}$ foot?

30. Assuming that wrought iron is 8 times as heavy as water, what is the weight of a cubic foot of iron, a cubic foot of water weighing 1000 ounces?

31. A room is 20 feet long, 15 feet wide, 9 feet high. How many square yards in the walls?

32. Two partners contribute \$400 and \$500, respectively. How will they divide profits of \$180?

33. How many inch cubes can be placed in a box 8 inches long, 5 inches wide, and 3 inches deep?

34. Find the cost of 60 yards of muslin at $11\frac{1}{2}$ cents per yard.

35. If each shingle is laid so that $\frac{1}{3}$ foot by $\frac{1}{3}$ foot is exposed to the weather, how many shingles will be needed for 100 square feet, not counting waste?

36. How many pounds equal 1000 ounces?

37. At 27 cents per square yard, what will it cost to plaster the ceiling of a room 20 feet long, 15 feet wide?

38. A bushel of corn in the ear measures $2\frac{1}{5}$ cubic feet. How many bushels will occupy 3300 cubic feet?

39. How many yards of carpet $\frac{3}{4}$ yard wide will be needed to cover a floor 15 feet by 9 feet?

40. When a man 6 feet tall casts a shadow of 4 feet, the shadow of a steeple is 48 feet. How high is the steeple?

41. How many board feet are there in 10 boards each 12 feet long, 8 inches wide, 2 inches thick?

42. If a 1-foot iron cube weighs 500 pounds, what will be the weight of a 2-foot cube?

43. What is the smallest whole number by which 24 must be multiplied to produce a perfect square?

44. A man of 48 is 3 times as old as his son. What will be the ratio of their respective ages in 16 years?

45. What is the cost of an acre of land at the rate of \$21.75 for $\frac{3}{4}$ acre?

46. If taxes are \$1.50 per \$1000, what must be paid on property worth \$4800?

47. What is the smallest number that will have a remainder of 1 when divided by 3, by 4, or by 5?

48. A can do a piece of work in 4 days, B can do it in 6 days. How many days will it require for both to do the work ?

49. A man owing \$1500 has only \$1000. How much should M receive to whom \$240 is due ?

50. Divide 60 into 3 numbers proportional to 3, 4, and 5.

51. Philadelphia has the time of 75° west longitude, and Berlin that of 15° east longitude. What is the time at Philadelphia when it is 3 P.M. at Berlin ?

52. How many acres are there in a field 120 rods long, 80 rods wide ?

53. $\$6.24 \div 1.04 = ?$

54. Three 25ths is what part of three 5ths ?

55. How many blocks 1 inch \times 1 inch \times 1 inch will be required to make a pile 1 foot \times 1 foot \times 1 foot ?

56. Find the solid contents of a cube, the area of one face being 100 square inches.

57. A bale of hay will last a horse 3 weeks or a calf 6 weeks. How long will it last both ?

58. What is the value of a pile of 4-foot wood 16 feet long, 4 feet high, at \$4.50 per cord ?

59. How many board feet in a piece of timber 18 feet long and 10 inches square ?

60. Find the convex surface of a pyramid whose base is 5 feet square and whose slant height is 6 feet.

61. What will be the cost at \$1 per 1000 cubic feet for the gas burned during November, if 5 lights, each consuming 12 cubic feet per hour, are used per night for an average of 4 hours each ?

62. What does a dealer receive for a 2240-pound ton of coal sold for \$5 per ton of 2000 pounds ?

63. How many feet of board will be required for a tight fence 6 feet high inclosing a lot 25 feet \times 100 feet?

64. How long will it take a train to go 30 miles at the rate of 50 miles an hour?

65. By selling a house for \$3200, the owner lost $\frac{1}{5}$ of its cost. What did he pay for it?

66. What is the ratio of the area of a 20-foot square to that of a 30-foot square?

67. Four fifths of what number equals 96?

480. Miscellaneous Written Problems.

1. A lot 44 feet by 110 feet sells for \$850. What would an acre cost at the same rate?

2. Find the cost of 7 one-inch boards 16 feet long, 8 inches wide, at \$23 per M.

3. If a plow turns a furrow 10 inches wide, what is the total length of all the furrows turned in plowing a 10-acre field?

4. How many steps of 2 feet 4 inches each will it take to measure a mile?

5. A floor 20 feet 4 inches by 16 feet 8 inches is laid with tiles 4 inches square. How many tiles are used?

6. A man left \$3600 by will to be distributed among three servants in proportion to their times of service, which were $7\frac{1}{2}$, $10\frac{1}{2}$, and $15\frac{3}{4}$ years, respectively. What was the share of each?

7. Show how a concrete quotient must be obtained. If the multiplicand be concrete, what must be the unit of the product? Explain why a concrete number cannot be separated into two like factors.

8. A ladder 50 feet long is so placed in the street that without being moved at the foot, it will reach a window on

one side 40 feet and on the other side 48 feet from the ground. How wide is the street?

9. In what time will 8 masons build a wall 84 feet long, working 10 hours a day, if 12 masons build a wall 96 feet long in 8 days, working 8 hours a day?

10. Find the cost of the lumber required for a floor 20 feet long, 18 feet wide, and 2 inches thick, at \$15 per M.

11. Three pipes will fill a cistern in 4 hours. The first would require 12 hours to fill it and the second 8 hours. How long would it take the third to fill it?

12. A pile of wood 12 feet long, 6 feet high, and 4 feet wide is sold for \$11.25. At what price per cord was it sold?

13. At the time a staff 6 feet above the ground casts a shadow measuring 8 feet, the shadow of a steeple measures 144 feet. How high is the steeple?

14. Divide \$2380 between A and B so that $\frac{2}{3}$ of A's share will equal B's.

15. A merchant sold $\frac{3}{8}$ of a quantity of cloth at $\frac{5}{8}$ of its cost, and the remainder at cost, thereby gaining \$7.20. What did the cloth cost?

16. If 24 men in 5 days can build a wall 72 rods long, how many rods of wall can 15 men build in 6 days?

17. How many cubic feet of stone will be required to build a wall 2 feet thick and 4 feet high on the outside of a plot 20 feet square, making no allowance for mortar?

18. A creamery buys 1800 quarts milk per day at a cost of 2¢ per quart. From 100 quarts of milk are obtained 7 pounds butter, which sells at 25¢ per pound, 12 quarts butter-milk sold at 3¢ per gallon, and 84 quarts skimmed milk sold at 5¢ per gallon. The daily expenses for labor, etc., are \$7.50. What is the daily profit?

19. Divide \$10,450 into two parts having a ratio of 2 to 3.

20. How many rods of fence will be required to inclose a field in the form of a right-angled triangle containing 40 acres, the base measuring 128 rods?

21. A dairyman has 10 cows averaging 840 pounds each. How many tons of hay will they consume in a year, if each receives daily $\frac{1}{30}$ of its weight of hay?

22. A tank can be filled by pipe *A* in 12 hours and pipe *B* in 9 hours. Pipe *C* will empty it in 8 hours. How long will it require for *A* and *B* to fill it, if *C* is closed? If all three are open? How long will it take to empty it when full, if *A* and *C* are open? If *B* and *C* are open?

23. What is the value of a pile of wood 12 feet long, 4 feet wide, and $8\frac{1}{2}$ feet high, at \$4.50 per cord?

24. A cow gives milk for 300 days per year, yielding 16 quarts daily for 35 days, 12 quarts daily for 65 days; and 5 quarts daily for the remainder of the time. If one half the milk is sold at 12¢ per gallon and the other half at 16¢ per gallon, how much is received for the milk during the year?

25. How many miles, exclusive of turns, would a team be required to travel to cut 10 acres of grain with a reaping machine, if the machine cuts a strip 66 inches wide?

NOTE. An acre is 66 inches wide by how many miles long?

26. A farmer had 320 bushels of potatoes to sell. He sold one half of them in the fall at 60¢ per bushel. The other half he sold in the spring by weight, after they had lost $\frac{3}{10}$ of the original weight, at 75¢ per bushel. How much more did he receive for those sold in the spring?

27. An exporter bought through an agent 1500 tierces of lard at \$7.11 $\frac{1}{2}$ per tierce. The agent charged him 2 $\frac{1}{2}$ ¢ per tierce for buying. What was the total cost of the lard, and the price per pound, a tierce containing 340 pounds?

28. At \$ 20 per M, what will be the cost of a piece of timber 30 feet long, 14 inches square, and a plank 20 feet long, 8 inches wide, $2\frac{1}{2}$ inches thick?

29. A cistern which holds 100 gallons can be filled from a pipe in 25 minutes and emptied by a waste pipe in 45 minutes. If both are opened together, how long will it take to fill the cistern, and how much water will have been wasted by that time?

30. An agent sold for a dealer 600 bales of cotton, averaging 500 pounds per bale, at 11.04¢ per pound. How much did the dealer receive for the cotton, if the agent deducted \$ 5 per 100 bales for selling it?

31. A fast train starting at 8:30 A.M. reaches its destination at 4:45 P.M., making 3 stops of 3 minutes each. Find the average speed per hour of running time, the distance being 440 miles.

32. If a team makes a furrow 10 inches wide and 9.9 miles long in a day, what part of an acre is contained in the furrow?

33. If $7\frac{1}{2}$ gallons of water occupy one cubic foot, how many gallons of water would a cylindrical boiler contain whose diameter is 24 inches and whose height is 7 feet?

34. A freight car 30 feet long, 9 feet wide, is filled with wheat to a depth of 8 feet. How many bushels of wheat does the car contain, if one bushel of grain occupies $1\frac{1}{4}$ cubic feet? Find the weight of the wheat at 60 pounds to the bushel.

35. If 24 men each working 10 hours a day do two-thirds of a piece of work in 28 days, how long should it take 20 men working 8 hours a day to do the whole work?

36. How many acres are there in a rectangular field 402 rods 2 feet long and 120 yards wide?

37. A pile of wheat 5 feet high lies in the corner of a barn, each point of the base being 7 feet from the corner.

How many bushels are there in the pile, if a bushel equals $1\frac{1}{4}$ cubic feet?

One fourth of a cone 5 feet high, radius 7 feet.

38. A and B were doing a piece of work which both together could finish in 15 days. After working 6 days, A leaves, and B completes it in 36 days. How many days would B require to do the whole work?

39. The distance between two towns is 1584 kilometers. What is the distance in miles?

1 kilometer = 1000 meters. 1 meter = 39.37 inches.

40. A map representing an area of 72,000 square miles measures 12 inches by 15 inches. On what scale is it drawn?

41. A farmer hired two men and two boys to harvest sugar beets, paying them \$94.50 for their work. If they worked $22\frac{1}{2}$ days, how much did each receive per day, the boys being paid one half as much as the men?

42. Write one thousand forty-five and nine millionths. Write seven and nine ten-thousandths.

43. How much wheat will it take to seed a field 76 rods wide by 105 rods long, at the rate of $1\frac{1}{2}$ bushels to the acre?

44. Mr. Lally buys 3 horses from Mr. Tully at \$125. In payment he delivers to the latter 10 loads of hay averaging 1875 pounds each, for which he receives credit at the rate of \$12 per ton, and 5 jars of butter averaging 9 pounds each, for which he is credited at the rate of 24¢ per pound. Make out Mr. Tully's statement of the account.

45. A cistern 4 meters by 25 decimeters by 2 meters is supplied by a pipe which discharges 50 liters per minute. How long will it take to fill the cistern?

46. The foot of a ladder is 32 feet from the base of a building and the top of the ladder just reaches a window 60 feet from the ground. How long is the ladder?

47. If a company of 200 men consumes 35 barrels of flour in 7 weeks, how long will it take 400 men to consume 40 barrels?

48. How many board feet in 6 pieces of timber, each 20 feet long and 10 inches square?

49. The sum of three numbers is 940. The first equals $\frac{5}{9}$ of the second and the second equals $\frac{7}{10}$ of the third. Find the numbers.

50. In a tank 11 feet by 7 feet the oil was 4 feet deep. How many gallons were drawn when the oil measured 2 feet 9 inches in depth?

51. A country is 480 miles long and 360 miles wide. Give the dimensions of the paper on which is drawn the map on the scale of $\frac{1}{8}$ inch to the mile, with a border of 2 inches.

52. The population of a certain city is $\frac{1}{5}$ greater each year since 1902 than it was the previous year. In 1905 the population was 108,000. What was the population in 1902? In 1907?

53. What are the dimensions of a rectangle whose area is 2187 square feet and whose length is three times its breadth?

NOTE. Make a diagram. Divide the rectangle into equal squares.

54. Divide \$269.50 among four persons in the ratio of 30, 20, 15, and 12. In the ratio of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.

55. Simplify $(\frac{2}{3} \times \frac{5}{12} \times 4\frac{3}{4}) \div (1\frac{1}{2} + 4\frac{5}{6})$.

56. Find the value of the following:

$$(0.139 \times 28) + (42 \times 0.002) + (6 \times 0.004) - (0.05 \times 20).$$

57. A pile of wood containing $67\frac{1}{2}$ cords is 270 feet long and 4 feet wide. How high is it?

58. If from $18\frac{3}{4}$ yards of cloth $4\frac{1}{8}$ yards are cut, what fraction of the piece is left?

59. A square court is paved with 48,841 stones, each one foot square. Find the dimensions of the court.

60. A can do $\frac{1}{2}$ of a piece of work in a day and B can do $\frac{1}{3}$ of it in a day. They work together and receive \$6 for the job. How long does it take them to do the work, and how much should each receive?

61. Find the cost of 12 planks, each 10 feet long, 12 inches wide, and 3 inches thick, at \$75 per M.

62. Find the result of the following:

$$[(72.2 \div 10) - 2] \div (0.5 \div 1.6) + [2.125 \div (1.75 - .05)].$$

63. What is the cost of a board 24 feet long, 23 inches wide at one end and 17 inches wide at the other, and $1\frac{1}{2}$ inches thick, at \$30 per M?

64. Extract the cube root of 15.625.

65. Reduce $\frac{9}{11}$ mile to rods, yards, and feet.

66. A farmer kept 450 bushels of potatoes during the winter, losing .1 by decay. He sold the remainder in the spring for 80 cents a bushel, receiving \$54 more than he could have obtained in the fall. What was the price in the fall?

67. After spending $\frac{1}{5}$ of his money, $\frac{1}{10}$ of the remainder, and $\frac{1}{10}$ of what he then had, a man has \$142.56. What had he at first?

68. How many barrels of water, of $31\frac{1}{2}$ gallons each, will fall on a garden 5 rods by 6 rods, during a shower in which the fall of rain is three fourths of an inch?

69. A circular park has a road around it. The outer circumference of the road is 440 yards, and its width is 60 feet. Find the area of the park exclusive of the road.

70. A man agrees to work for \$3 a day and his board, and to pay \$1 a day for his board when idle. At the end of 30 days he receives \$38. How many days did he work?

71. A ship's chronometer showing Greenwich time was 40 minutes slow Saturday noon and 15 minutes fast the following Tuesday noon. In what longitude was the ship at each observation?

72. A and B together erect a shed at a cost of \$ 82.50, A paying \$ 22.50, and B the remainder. They sold it afterwards for \$ 27.50. How much should each receive?

73. At \$ 1.50 per rod, what is the difference in the cost of fencing 10 acres in the form of a square, and the same area as a rectangle 20 rods wide?

74. A man travels 48 miles due north, then 48 miles due east, then 28 miles due south. How far is he then from his starting point?

75. A person failing in business owes \$ 4500 and has property worth \$ 1800. How much will a creditor receive whose claim is \$ 275?

76. How many cubical blocks, each 2 inches by 3 inches by 4 inches, will be required to fill a space 2 feet by 3 feet by 4 feet?

77. Simplify $\frac{5\frac{2}{3} \div 3\frac{3}{5}}{2\frac{3}{8} - 1\frac{1}{2}} \times \frac{3\frac{1}{4} \div 2\frac{2}{3}}{4\frac{2}{7} \times \frac{1}{16}}$.

78. Find the cost of 25 boards at \$ 60 a thousand, each board being 14 feet long and 14 inches wide.

79. What is the diagonal of a rectangle whose sides measure 32 rods and 60 rods, respectively?

80. Find the capacity, in gallons, of a rectangular cistern 10 feet 6 inches by 4 feet 6 inches by 6 feet 3 inches.

81. A room 24 feet \times 28 feet and 9 feet high has 3 doors, each 3 feet \times 8 feet, and 3 windows, each 3 feet \times 6 feet. Find the cost of plastering the walls and the ceiling at 5¢ a square yard, deducting one half the area of the openings.

82. During a month of 21 school days, there were 14 girls and 11 boys in attendance. Three girls were absent

1 day each, 4 boys $1\frac{1}{2}$ days each, and 2 boys 2 days each. Find the average attendance of (a) the girls, (b) the boys, (c) the school.

83. A man owing \$ 1800 paid his debt in six years and had \$ 900 left by saving $\frac{5}{17}$ of his salary. What was his annual salary?

84. A certain street one half mile long and 6 rods wide is excavated to an average depth of $2\frac{1}{2}$ feet. Find the cost at 70 cents a cubic yard.

85. To make a certain grade of concrete, there are used 2 parts of lime, 1 of cement, and 6 of broken stone. How many cubic feet of each are used in building a wall $36' \times 9' \times 1\frac{1}{2}'$?

86. If milk is 1.03 times as heavy as water, what is the weight of the milk that fills a can 28 inches in diameter and 3 feet high?

87. A square prism 11 feet high has a volume of 4851 cubic feet. Find one side of the base.

88. Find the cost of a pile of 4-foot wood 40 feet long and 6 feet high, at \$ 5 per cord.

89. A square field containing 10 acres is divided into four equal square fields. Find the number of rods of fence required to inclose them.

90. Find the value of $(0.125)^2 \times (0.32)^3$.

91. Simplify $\frac{8\frac{7}{8} - 7\frac{6}{7} + 5\frac{5}{6} - 4\frac{4}{5}}{9\frac{9}{10} - 8\frac{1}{15} + 7\frac{7}{8} - 6\frac{6}{7}}$.

92. What will be the cost of digging a ditch 30 rods long, 3 feet deep, 6 feet wide at the top and 4 feet wide at the bottom, at 8 cents a cubic yard?

93. What is the difference between the length of a fence around a circular piece of land comprising 43,681 square

rods, and the length of a fence around the same quantity of land in the form of a square ?

94. A spherical shell whose internal diameter is 7 inches is filled with water. Its contents are poured into a cylindrical vessel whose internal radius is 7 inches. Find the depth of water in the cylinder.

CHAPTER VIII.

GENERAL REVIEW.

NUMBERS.

481. **Arithmetic** is defined as the science of numbers; the art of computation by figures.

482. A **number** may be used to give the length of a line, the size of an angle, the weight of a quantity of sugar, the area of a field, etc., or to state how many sheep there are in a flock, people in a crowd, etc.

483. The number of sheep in a flock, for instance, is obtained by *counting*; as is the number of pupils in a room, the number of apples on a plate, and the like.

484. The length of a table is ascertained by *comparing* it with a known length taken as a standard. A foot rule, for instance, is employed, and if the length of the table contains the length of the ruler exactly four times, the number *four* expresses the length of the table in terms of the **unit**, the foot.

The number *four* obtained in this case is called a **whole number**, because it contains only *entire units*. A whole number is also called an **integer**.

485. A line shorter than a foot may be measured by employing the divisions of the foot rule. A folding rule contains four divisions; and if the length of a line is equal to the length of three of the divisions, the line is said to be three fourths of a foot long.

The number *three fourths* is called a **fraction**, because it does not indicate entire units.

486. A length equal to four times the foot rule and three of the divisions, is said to be four and three quarters feet long.

The number *four and three quarters*, which contains a whole number and a fraction, is called a **mixed number**.

487. In measuring this last line, the inch divisions, marked on the foot rule, might be employed as units in determining the length of the portion of the line remaining after the rule was employed four times in ascertaining the whole number of feet. This remaining portion contains nine of the inch divisions, and the line is said to measure four feet nine inches.

The expression *four feet nine inches* is called a **compound number**, because it contains units of different magnitudes, but of related kinds. Six pounds and eight hours do not form a compound number.

488. Things that can be measured are called **magnitudes** or **quantities**.

A magnitude is said to be *continuous* when it forms a whole without presenting any division ; as, the area of a field, the time taken by a trip, or the weight of a body. A magnitude is said to be *discontinuous* when it is made up of distinct like objects, or of objects merely having the same name ; such as, the steps of a flight of stairs, the windows of a house, the cattle in a drove.

489. A continuous magnitude is measured by ascertaining the number of times it contains a standard taken as a unit. A discontinuous magnitude is measured by counting the objects composing it. Each of the names employed in counting represents a *number*.

490. An **abstract number** is one in which the kind of unit is not expressed ; in a **concrete number** the unit is indicated.

Thus: five, four and three quarters, eight, are abstract numbers; eighteen horses, six hours, two feet eight inches, are called concrete numbers.

491. Concrete numbers in which the units are those employed in measuring continuous magnitudes are called **denominate numbers**; such as four acres, twenty-five degrees, seven miles.

NOTATION AND NUMERATION.

Formation of Numbers.

492. It has been found possible to name all numbers by the use of comparatively few words. One, two, three, etc., to ten; hundred, thousand, million, billion, with a few others, are all that are required separately or in combination to express any number. Eleven and twelve are old forms, meaning ten and one, ten and two; the suffix "teen" means "and ten" when united to four, five, etc., to nine. The suffix "ty" means "times ten" when combined with six, seven, eight, and nine. The first syllable of twenty, thirty, forty, fifty, is a modification of two, three, four, and five, respectively.

493. The nine numbers following twenty are twenty-one, twenty-two, etc., to twenty-nine. Those following thirty, forty, etc., to ninety are formed in the same way.

494. The number after ninety-nine is one hundred. Numbers to one hundred ninety-nine are indicated by adding after one hundred the numbers from one to ninety-nine. Those after two hundred, three hundred, etc., to nine hundred ninety-nine follow the same rule.

495. After nine hundred ninety-nine comes one thousand. The thousands are consecutively numbered to nine hundred ninety-nine thousand, the next thousand being called a million, which is equal to one thousand thousands. One thou-

sand millions is called a billion; one thousand billions, a trillion; etc.

In some countries a billion is a million millions.

Writing Numbers.

496. The necessity of expressing in writing a number of any size in the shortest possible way, led to the introduction of characters to represent a few important numbers, the other numbers being formed by combining these characters.

497. Roman Notation. The Romans employed seven letters, I, V, X, L, C, D, and M, to represent one, five, ten, fifty, one hundred, five hundred, and one thousand, respectively. Two was written II; and three, III. Four was written IV. Six, seven, and eight were written VI, VII, and VIII, respectively, nine being written IX. The numbers from ten to nineteen were written by placing after X the letters representing the numbers from one to nine, inclusive. Twenty and thirty were written XX and XXX, forty being XL.

498. Roman numbers are written by expressing the number of hundreds by an equivalent number of C's, substituting, however, a D for five C's; by expressing the tens by X's, substituting an L for five X's; and the number of ones by I's, substituting a V for five I's. Four and nine may also be expressed as IV and IX; forty and ninety as XL and XC; four hundred and nine hundred as CD and CM.

499. Arabic Notation. By the Arabic system, all numbers are expressed by ten characters, as follows:

0,	1,	2,	3,	4,	5,	6,	7,	8,	9
naught	one	two	three	four	five	six	seven	eight	nine

The naught (cipher or zero) has no value except to fill a vacant place; the others are called **significant figures**.

500. To express ten, two figures are used, 10. This means 1 ten, no units; the figure in the second place, counting from the right, represents the number of tens, and that in the first place, the number of units. The succeeding tens are written: 20, 30, etc., to 90.

To express hundreds, a figure in the third place is required; thus, 300, 500, 700, 900. Tens and units replace the zeros to form intermediate numbers. Thousands are represented by figures in the fourth, fifth, and sixth places; thus, 2,000, 14,000, 256,000, etc.

Figures in the seventh, eighth, and ninth places represent millions; etc.

501. Simple and Local Value. In the number 60,341, the 1, being in the units' place, has a local, or place, value of one. The figure 4, being in the tens' place, has a local value of 40. The figure 3, being in the hundreds' place, has a local value of 300. The figure 6, being in the fifth place, has a local value of 60,000. The zero has no value; it merely serves to show that there is no figure representing thousands below ten thousand.

502. The local value of a significant figure increases tenfold as it is moved from right to left, its local value in the first place being the same as its simple value.

503. Notation of Decimal Fractions. Our system of writing numbers is called the **decimal system**, from the Latin word *decem*, meaning ten, because of this tenfold increase in the local value of the figures from right to left. Fractions made up of tenths, hundredths, thousandths, etc., of a unit, may also be represented by figures placed to the right of the units' place, and separated therefrom by a period. Such fractions are called **decimals**.

Thus, we write the mixed number seventy-five and twenty-three hundredths 75.23, the 7 representing 70, the 5 representing 5, the 2 representing 2 tenths, the 3 representing 3 hundredths.

504. The decimal 406 ten thousandths is written .0406. There being no whole number, the decimal point is first written; and as ten-thousandths comprises four decimal places, a cipher must precede the first significant figure of the decimal.

505. Notation of Common Fractions. When it is said that a certain line measures five sixths of a foot, it means that the standard unit, the foot, was divided into six equal parts called sixths, and that the line was equal in length to five of the divisions.

506. To write five sixths, we place 5 above a line and 6 below, thus, $\frac{5}{6}$. A fraction written in this form is called a **common fraction**. The number below the line is called the **denominator**; the one above the line is called the **numerator**.

507. In decimals it is unnecessary to write the denominator, as the number of places following the decimal point indicates that the denominator is 1 followed by as many ciphers as there are such places.

508. In any fraction the denominator shows the size of the *fractional unit*; that is, the number of parts into which the standard unit has been divided; the numerator shows the *number* of the fractional units. The smaller the denominator, the larger is the fractional unit; $\frac{1}{2}$ being larger than $\frac{1}{4}$, $\frac{1}{6}$, etc.

509. In writing a *mixed number*, the common fraction immediately follows the whole number. It forms a portion of the unit, not being counted separately in determining the local value of the figures composing the whole number.

510. A *compound number*, such as 3 pounds 10 ounces, is written with no mark between the different related units. The denominations, however, are generally abbreviated thus: 3 lb. 10 oz.

511. A *per cent* is written by placing the sign % after the number.

512. Numeration of Integers. Numbers are separated into periods of three figures each, beginning at the right: the first period representing units; the second, thousands; the third, millions; as shown below. Each period is divided into units, tens, and hundreds. The number is read by reading each period separately, beginning at the left, and giving the name of each period as it is read, but omitting the name of the units' period.

NAMES OF PERIODS :	Millions			Thousands			Units		
NAMES OF PLACES :	{ Hundred-millions Ten-millions Millions }			{ Hundred-thousands Ten-thousands Thousands }			{ Hundreds Tens Units }		
THE NUMBER :	1	2	3	4	5	6	7	8	9
ORDERS :	9th	8th	7th	6th	5th	4th	3d	2d	1st

The number is 123 million, 456 thousand, 789.

It is composed of 9 units of the 1st order, 8 of the 2d, 7 of the 3d, 6 of the 4th, 5 of the 5th, 4 of the 6th, 3 of the 7th, 2 of the 8th, 1 of the 9th.

513. In reading a number containing ciphers, read only the significant figures.

308,000,020 is read "three hundred eight million, twenty."

514. Reading Decimals. A decimal is expressed in words by reading it as a whole number followed by the denomination of the last figure, counting from left to right, as shown on the next page.

.				
	tenths	hundredths	thousandths	ten-thousandths
.0	2	0	4	6

The decimal .02046 is read "2046 hundred-thousandths."

515. In reading a whole number and a decimal, the word *and* is generally used between the former and the latter, being omitted both in reading the whole number and in reading the decimal.

Thus, 8034.0456 is read "8 thousand 34 *and* 4 hundred 56 ten-thousandths."

516. NOTE. To avoid confusion, however, it is advisable to employ the word *units* in such combinations as the following: 300.008, which should be read "300 units and 8 thousandths." It may also be read "300 and decimal 8 thousandths."

517. Reading Fractions. A fraction is read by reading the numerator, followed by the denominator read as a fractional unit. $\frac{1}{2}$ is read as one half; $\frac{1}{4}$, as one quarter; $\frac{1}{3}$, as one third. In other cases, the fractional unit is expressed by the suffix "th" or "ths." A unit numerator is frequently read as "a"; $3\frac{1}{2}$ being called 3 and a half.

NOTE. The word *and* is employed between the whole number and the fraction in reading mixed numbers.

518. Reading Per Cents. Per cents composed of whole or mixed numbers are read by using the words *per cent* after the number.

The following illustrations will show the usual business method of reading fractional per cents:

$\frac{1}{2}\%$ is read "one half of one per cent."

$\frac{3}{10}\%$ is read "three tenths of one per cent."

$.5\%$ is read "five tenths of one per cent."

519. Exercises, Oral and Written.

1. Write a number containing 6 units of the second order, 4 units of the third order, and 8 units of the sixth order.
2. What number contains 4 units of the fifth order, 7 units of the fourth order, and 2 units of the first order?
3. Write 14 and 77 hundred-thousandths.
4. In the number 360,912, what is the place (local) value of the first two figures, reading from the left? What is the place value of the sixth figure? What is the ratio of the place value of the second figure to the place value of the first figure? What is the place value of the first two figures as compared with the last two? Of the second two compared with the last two? Of the fourth compared with the last two?
5. Answer the foregoing questions with relation to the number 3609.12.

REDUCTIONS.**520. Preliminary Exercises.**

1. How many units of the first order are there in 3 units of the second order?
2. In 5 units of the fourth order?
3. In 7 units of the sixth order?
4. How many tenths in 1 unit? In 2 units?
5. How many sixths in 1? In 2?
6. How many ounces in 2 pounds? In $\frac{1}{2}$ pound?
7. How many fourths in 2? In $2\frac{1}{4}$?
8. How many inches in 2 feet? In 2 feet 3 inches?
9. How many hundredths in 2?
10. What per cent of a number equals 3 times the number?

521. Reduction to Higher Terms; Reduction Descending. Changing a given number of fourths to an equivalent number of twelfths, of hundreds to tens, of tenths to thousandths, of pounds to ounces, of acres to square rods, and the like, involves in each case a change of unit. In the case of the fraction, it is called **reduction to higher terms**, because the terms of the fraction are larger; in the other instances it is more properly called **reduction descending**.

In changing 3 fourths to twelfths, for instance, we multiply 3 twelfths, the equivalent of 1 fourth, by 3, the number of fourths, which gives 9 twelfths as the result. To change 2 pounds to ounces, 16 ounces, the equivalent of 1 pound, is multiplied by 2, the number of pounds, which gives 32 ounces as the result.

522. A whole number is changed to a fraction by multiplying the denominator of the fraction by the whole number, the denominator indicating the number of fractional units in the unit of the whole number. In changing a mixed number to a fraction having the same denominator as that of the fraction, the whole number is first reduced to the fraction, and the fractional part of the mixed number is added to it.

To reduce $17\frac{4}{5}$ to a fraction, 17 is changed to fifths by multiplying 5 by 17, which gives 85 fifths. To this is added 4 fifths, making 89 fifths, written $\frac{89}{5}$.

In practice, 5, the smaller number is taken as the multiplier, and the numerator, 4, is added in at the time the multiplication is performed.

523. A fraction in which the numerator is equal to the denominator or greater than it, is called an **improper** fraction. A fraction whose numerator is less than the denominator is called a **proper** fraction.

524. Oral Exercises.

1. Reduce $22\frac{2}{3}$ to an improper fraction.
2. Change $2\frac{1}{4}$ to eighths.

3. How many ounces in 10 lb. 13 oz. ?
4. Change 10 acres to square rods.
5. How many tens in 316 thousands ?
6. How many units of the second order are equivalent to 8 units of the fifth order and 3 units of the third order ?
7. Read 3 million as tens. As hundreds. As thousands. As tenths.
8. Change $\frac{1}{4}$ to hundredths.
9. Express $\frac{1}{8}$ as a per cent.
10. Express $\frac{1}{8}$ as a decimal.
11. Reduce .125 lb. to ounces.
12. How many sq. rd. in $\frac{3}{4}$ acre ?
13. Change $\frac{2}{3}$ to sixths. To ninths. To 12ths. To 24ths. To 30ths. To 120ths.
14. How many thirds in 31 ?
15. Change 42 to an improper fraction having 10 for a denominator.

Reduction to Lower Terms; Reduction Ascending.

525. Preliminary Exercises.

1. Reduce 316,000 to hundreds.
2. Change 1600 sq. rd. to acres.
3. Reduce 173 ounces to pounds and ounces.
4. Reduce $\frac{42}{8}$ to an improper fraction in lower terms.
5. Change $\frac{68}{3}$ to a mixed number.
6. How many units of the fifth order are equal to 800 units of the third order ?
7. Change $\frac{23}{3}$ to a whole number.
8. Reduce 14 ounces to the fraction of a pound. To the decimal of a pound. To the per cent of a pound.

9. Reduce $\frac{8.0}{120}$ to 60ths. To 30ths. To 15ths. To lowest terms.

10. What fraction of a day is 4 hr. 48 min. ?

526. A fraction is said to be expressed in *lowest terms* when the numerator and the denominator are the smallest possible whole numbers, the unit of the denominator in this case being the largest one possible. When the denomination of a compound number is changed from a smaller unit to a larger one, the process is called **reduction ascending**.

527. To change a fraction to lower terms, the numerator and the denominator are divided by the same numbers.

Thus, $\frac{875}{1000} = \frac{175}{200} = \frac{35}{40} = \frac{7}{8}$, the change in each case being effected by dividing both terms by 5. When the denominator is divided by 5, the fractional unit is made 5 times as large ; one fifth of the number of units in the numerator must be taken to keep the fraction equal to the original. 875 thousandths = 175 two-hundredths = 35 fortieths = 7 eighths. In the same way, 875 mills = 87 cents 5 mills = 8 dimes 7 cents 5 mills. In this case, however, there are three different denominations, the result being a compound number.

528. Oral Exercises.

1. Change 4900 sq. rd. to acres, etc.
2. Reduce $\frac{3.50}{420}$ to lowest terms.
3. Change $4\frac{1}{3}$ hours to the fraction of a day.
4. Reduce 46 days to weeks and days.
5. What % of a day is $7\frac{1}{3}$ hours ?
6. Change 1 ft. 6 in. to the fraction of a yard.
7. What decimal of a pound is 17s. 6d. ?
8. Change $\frac{1.8}{27}$ to lowest terms.
9. What fraction of a cubic yard is 18 cubic feet ?
10. What decimal of a mile is 40 rods ?

FACTORS AND MULTIPLES

529. The **factors** of a number are those numbers of which the given number is the product.

Every number is, of course, divisible by itself and 1; these will, therefore, be omitted from consideration in determining the factors of a number.

Preliminary Exercises.

530. Find the factors of the following numbers:

- | | | | |
|-------|-------|--------|--------|
| 1. 14 | 5. 35 | 9. 51 | 13. 74 |
| 2. 21 | 6. 39 | 10. 57 | 14. 77 |
| 3. 25 | 7. 46 | 11. 65 | 15. 82 |
| 4. 33 | 8. 49 | 12. 69 | 16. 85 |

531. The following numbers have three factors each; what are they?

- | | | | |
|-------|-------|--------|--------|
| 1. 8 | 5. 66 | 9. 99 | 13. 50 |
| 2. 12 | 6. 78 | 10. 63 | 14. 75 |
| 3. 30 | 7. 20 | 11. 28 | 15. 70 |
| 4. 42 | 8. 45 | 12. 44 | 16. 98 |

532. A **prime number** is a number that cannot be separated into integral factors each greater than 1; such as 1, 2, 3, 5, 7, etc. Numbers that are not prime are called **composite numbers**; such as 4, 6, 8, 9, 10, etc.

533. **Divisibility of Numbers.** — A number is divisible by 2 if it ends in 0, 2, 4, 6, 8. It is divisible by 5 if it ends in 0 or 5. A number is divisible by 3 or 9 if the sum of its digits (figures) is divisible by 3 or 9.

A number is divisible by 4 if its last two figures are divisible by 4; by 8, if its last three figures are divisible by 8. An even number is divisible by 6 if the sum of its digits is divisible by 3; etc.

534. Oral Exercises.

Which of the following numbers is divisible by 2, by 3, by 5, by 9?

- | | | |
|--------|---------|------------|
| 1. 120 | 5. 1825 | 9. 11,250 |
| 2. 475 | 6. 4684 | 10. 25,065 |
| 3. 570 | 7. 2346 | 11. 33,333 |
| 4. 243 | 8. 1234 | 12. 25,942 |

535. To find the prime factors of a number, divide it by any prime number; divide the quotient in the same manner; continue to divide until a quotient is obtained that is a prime number. The divisors and the last quotient will be the prime factors.

536. Written Exercises.

Find the prime factors of the following :

- | | | | |
|---------|---|---------|---------|
| 1. 630. | <i>Ans.</i> $2 \times 3 \times 3 \times 5 \times 7$. | | |
| 2. 798 | 4. 1000 | 6. 3672 | 8. 1836 |
| 3. 350 | 5. 1750 | 7. 5000 | 9. 1650 |

537. Oral Exercises.

What numbers will divide 36?

Since $36 = 2 \times 2 \times 3 \times 3$, any number that can be formed from these factors will be a divisor. They are 2, 3, 2×2 , 2×3 , 3×3 , $2 \times 2 \times 3$, $2 \times 3 \times 3$. *Ans.* 2, 3, 4, 6, 9, 12, and 18.

Find the divisors of :

- | | | | |
|-------|-------|-------|-------|
| 1. 24 | 3. 48 | 5. 60 | 7. 40 |
| 2. 50 | 4. 32 | 6. 45 | 8. 70 |

538. Give the numbers that will divide both :

The smaller number may be given as one divisor.

- | | | |
|--------------|--------------|--------------|
| 1. 24 and 36 | 4. 42 and 84 | 7. 21 and 63 |
| 2. 18 and 27 | 5. 16 and 56 | 8. 36 and 56 |
| 3. 32 and 60 | 6. 15 and 75 | 9. 27 and 81 |

539. The **greatest common divisor** of two or more numbers is the greatest number that will exactly divide each.

540. Find the greatest common divisor of 84, 126, and 168.

$$\begin{array}{r} 2 \overline{) 84 - 126 - 168} \\ 3 \overline{) 42 - 63} \\ 7 \overline{) 14 - 21} \\ \quad 2 - 3 \end{array}$$

$$2 \times 3 \times 7 = 42. \text{ Ans.}$$

Place the numbers in a row, and cancel 168. Since this is a multiple of 84, it will be divisible by any number that will divide 84.

Divide 84 and 126 by any prime number that is exactly contained in both, say 2. Divide the quotients in the same way, say by 3. Divide the new quotients by 7, the only prime number that is a common factor to both. The greatest common divisor is $2 \times 3 \times 7$, or 42, the product of the three common prime factors.

541. The chief use of the greatest common divisor in arithmetic is in the reduction of fractions to lowest terms. This, however, can generally be done more quickly by dividing both terms by a common factor, continuing the process until the fraction can no longer be reduced.

542. Written Exercises.

Reduce to lowest terms :

1. $\frac{600}{1320}$

4. $\frac{120}{264}$

7. $\frac{60}{135}$

2. $\frac{432}{1404}$

5. $\frac{484}{672}$

8. $\frac{108}{144}$

3. $\frac{540}{1260}$

6. $\frac{108}{288}$

9. $\frac{288}{384}$

543. Another method of finding the greatest common divisor of two numbers is by dividing the greater by the less, and the first divisor by the first remainder, the second divisor by the second remainder, and so on, until there is no longer a remainder, in which case the last divisor is the greatest common divisor.

This method is shown in the next example, in which $\frac{2916}{3072}$ is reduced to lowest terms by first finding the greatest common divisor of 2916 and 3072.

544. Reduce $\frac{2916}{3072}$ to lowest terms:

$$\begin{array}{r}
 1 \\
 2916 \overline{) 3072} \\
 \underline{2916} \quad 18 \\
 156 \overline{) 2916} \\
 \underline{156} \quad 18 \\
 1356 \\
 \underline{1248} \quad 1 \\
 108 \overline{) 156} \\
 \underline{108} \quad 18 \\
 48 \overline{) 108} \\
 \underline{96} \quad 4 \\
 12 \overline{) 48} \\
 \underline{48}
 \end{array}$$

Dividing 3072 by 2916 gives a remainder of 156, the new divisor. This is contained in 2916, 18 times with a remainder of 108, the next divisor. The next remainder, 48, becomes the divisor, leaving a remainder of 12. This, being contained exactly in 48, is the greatest common divisor of 2916 and 3072.

Dividing both terms of the fraction by 12, we get the equivalent fraction in lowest terms.

$$\frac{2916 \div 12}{3072 \div 12} = \frac{243}{256} \quad \text{Ans.}$$

545. Reduce to lowest terms:

1. $\frac{3535}{8989}$

3. $\frac{2047}{2231}$

5. $\frac{1649}{1989}$

2. $\frac{793}{1950}$

4. $\frac{153}{493}$

6. $\frac{323}{2193}$

546. A **multiple** of a number is produced by multiplying the given number by an integer. 12 is a multiple of 4, of 6, of 2, of 3. It is a *common* multiple of all of these numbers; 24, 36, 48 are common multiples of the foregoing numbers. The smallest number that will contain each is 12, which is called the **least common multiple**.

547. Preliminary Exercises.

1. Give three numbers that will contain 8 and 6.
2. What is the smallest number that will contain 8 and 6?
3. Find the least common multiple of 6, 8, and 12.
4. Give two common multiples of 10, 20, and 30.
5. What is the least common multiple of 10, 20, and 30?

548. Find the least common multiple of 6, 8, 12, 16, and 20.

$6 =$	Writing the numbers in a column, find the prime factors of each, canceling 6 and 8, which are contained in 12 and 16, respectively. The least common multiple of the remaining numbers must contain 2^4 , 3, and 5; it is therefore a product of these factors.
$8 =$	
$12 = 2 \times 2 \times 3 = 2^2 \times 3$	
$16 = 2 \times 2 \times 2 \times 2 = 2^4$	
$20 = 2 \times 2 \times 5 = 2^2 \times 5$	

$$2^4 \times 3 \times 5 = 240. \text{ Ans.}$$

549. Find the least common multiple of 12, 24, 36, 45, and 60.

$$\begin{array}{r} 2) \cancel{12} - 24 - 36 - 45 - 60 \\ \quad 2) \cancel{12} - 18 - 45 - 30 \\ \quad \quad 3) \cancel{6} - 9 - 45 - 15 \\ \quad \quad \quad 2 \quad - 15 \end{array}$$

$$2 \times 2 \times 3 \times 2 \times 15 = 360. \text{ Ans.}$$

Another method is to divide the numbers by any prime number that is a factor of at least two of the numbers, first canceling any of the numbers that is contained in any other; as, for instance, 12 in this example. Taking 2 as a divisor, the quotients are written underneath, 45 being brought down, since it is not a multiple of 2. 2 is again used as a divisor, giving quotients as shown, 45 being again brought down. 9 and 15 are canceled, being factors of 45, and 3 is taken as the next divisor. There being no common factor of the remaining numbers, 2 and 15, the least common multiple is obtained by multiplying together the three divisors and the two final quotients.

550. Written Exercises.

Find the least common multiple:

1. Of 9, 15, 18, and 27.
2. Of 2, 3, 4, 5, 6, and 7.
3. Of 125, 200, 250, and 300.
4. Of 240, 324, and 120.
5. Of 45, 60, 72, 18, and 12.

551. Miscellaneous Examples.

1. Give the prime factors of 3696.
2. Find three equal factors of 729.
3. What is the 5th power of four?
4. Find the greatest common divisor of 8947 and 10,603.
5. Find the least common multiple of 38 and 57.
6. What are the two equal factors of 2304?
7. Extract the square root of 4096.
8. What is the cube root of 19,683?
9. Find the 5th root of 3125.
10. Is 1537 a prime number?

THE FUNDAMENTAL PROCESSES.**ADDITION.**

552. The process of finding a number whose value is the combined values of two or more given like numbers is called **addition**. The numbers to be added are called **addends**. The number obtained by addition is called the **sum**, or **amount**. The sign of addition is $+$, called *plus*, which is a Latin word meaning "more."

553. The addends must be **like numbers**; that is, they must all be concrete numbers having the same unit, or abstract numbers. Concrete numbers of different related units must be reduced to the same unit before they can be added.

NOTE. A given number of boys and a given number of girls may be added to ascertain the number of pupils in a class; but they are considered as having the same unit, viz., pupils.

554. Addition of Integers.

Add 376, 1835, 29, 647, and 3324.

$$\begin{array}{r}
 376 \quad \begin{array}{l} 31 \text{ units} \\ 18 \text{ tens} \end{array} \\
 1835 \quad \begin{array}{l} 20 \text{ hundreds} \\ 4 \text{ thousands} \end{array} \\
 29 \\
 647 \\
 \hline
 3324 \\
 \hline
 6211
 \end{array}$$

The numbers are written in a row, the units, tens, hundreds, and thousands of each addend standing in the same column, respectively. The sum of the units' column is 31, which is equal to 3 tens 1 unit. Placing 1 in the column of units, 3 is added to the column of tens, making a total of 21 tens, equal to 2 hundreds 1 ten. Placing 1 in the column of tens, 2 is added to the column of hundreds, making a total of 22 hundreds, equal to 2 thousands 2 hundreds. Placing 2 in the column of hundreds, 2 is added to the column of thousands, making 6 thousands. Placing 6 in the column of thousands gives the result as 6211.

The small numbers written to the right show the totals of each column. Their sum gives the total of all the numbers.

555. In adding long columns, it is the practice of accountants to place alongside the sum of each column, including the number "carried." This enables them, in the case of an error in the total of one column, to obtain the correct result without going over all of the previous columns to ascertain the "carrying" figures.

$$\begin{array}{r}
 376 \quad \begin{array}{l} 31 \\ 22 \\ 6 \end{array} \\
 1835 \\
 29 \\
 647 \\
 \hline
 3324 \\
 \hline
 6211
 \end{array}$$

556. The pupils should employ as few words as possible in adding. In the example given, it is necessary in adding aloud, at the blackboard, for instance, to say only, 11, 20, 25, 31; 5, 9, 11, 14, 21; 5, 11, 19, 22; 5, 6.

557. To prove the correctness of the result in addition, the columns may be added *down* the second time, if the first result is obtained by adding *up*. Another plan is to divide a very long column into two portions, add each separately, and combine the sums. If the answer obtained in this way agrees with the first answer, the work may be presumed to be correct.

558. In adding two large numbers *mentally*, a pupil should add to the first number the hundreds of the second, then the tens, and lastly the units. Thus, the sum of 375 and 256 is most readily found by thinking, $375 + 200 + 50 + 6$.

559. Oral Exercises.

- | | | |
|-----------------|-----------------|-----------------|
| 1. $394 + 200$ | 2. $394 + 240$ | 3. $394 + 247$ |
| 4. $583 + 300$ | 5. $583 + 350$ | 6. $583 + 358$ |
| 7. $642 + 200$ | 8. $642 + 270$ | 9. $642 + 279$ |
| 10. $489 + 400$ | 11. $489 + 440$ | 12. $489 + 445$ |

348. **560. Addition of Decimals.** Decimals are
 25.6 added in the same way as integers, care being
 4.328 taken to have like units in the same column.
 506.27 The decimal point acts as a guide, a point being
 14. placed after each integral addend.
 898.198

561. Addition of Mixed Numbers.

Add $17\frac{3}{8}$, $14\frac{4}{5}$, $9\frac{2}{3}$, and $6\frac{1}{2}$.

$17\frac{3}{8} = 17\frac{45}{120}$ To find the sum of the fractions, they are
 $14\frac{4}{5} = 14\frac{96}{120}$ first reduced to fractions having a common
 $9\frac{2}{3} = 9\frac{80}{120}$ denominator.
 $6\frac{1}{2} = 6\frac{60}{120}$
 $\underline{48\frac{41}{120}}$ *Ans.*

In this example 120 is the least common denominator. The sum of the fractions is $\frac{281}{120}$, which is equal to $2\frac{41}{120}$. The fraction is written in its place, and 2 is carried to the units' column of the whole numbers.

The arrangement at the right facilitates the addition of the numerators by writing the common denominator above the numerators.

	120ths
$17\frac{3}{8}$	45
$14\frac{4}{5}$	96
$9\frac{2}{3}$	80
$6\frac{1}{2}$	60
<i>Ans.</i> $48\frac{41}{120}$	$\frac{281}{120} = 2\frac{41}{120}$

NOTE. The least common multiple of the denominators is the least common denominator of the fractions. This should be determined by inspection, if possible.

562. Written Exercises.

- $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{7}{8}$
- $12\frac{3}{4} + 7\frac{2}{5} + 95\frac{1}{8} + 3\frac{6}{7}$
- $42\frac{7}{10} + 15\frac{3}{100} + 62\frac{123}{1000} + 59$
- $321\frac{5}{18} + 105\frac{77}{100} + 64\frac{2}{9}$

563. Addition of Compound Numbers.

Add the following:

10 oz.	5 pwt.	17 gr.
3	18	20
	12	
5	3	
18 oz.	19 pwt.	13 gr.

Units of each denomination are placed in separate columns. Those of the lowest denomination are then added, and if the sum exceeds the number of units in the next higher denomination, it is reduced to the units of this denomination, and

the remainder is written under that column. The number of units of the higher denomination obtained by the reduction is "carried" to the next column, etc.

564. Written Exercises.

1. 14 sq. yd. 20 sq. ft. 100 sq. in., 15 sq. ft. 81 sq. in., 10 sq. yd. 18 sq. ft.

2. 15 A. 108 sq. rd., 24 A. 20 sq. yd., 15 A. 80 sq. rd. 15 sq. yd.

3. 17 bu. 3 pk. 2 qt., 58 bu. 7 qt., 35 bu. 2 pk. 7 qt., 11 bu. 4 qt.

4. 27 rd. 3 yd., 119 rd. 2 ft., 277 rd. 4 yd. 1 ft., 160 rd.

5. 46 gal. 2 qt. 1 pt., 33 gal., 23 gal. 3 qt. 1 pt., 96 gal. 1 qt.

SUBTRACTION.

565. Considering **subtraction** as the reverse of addition, it may be defined as an operation to find the remaining quantity, when the sum of two like quantities is given, and one of the quantities.

566. Subtraction of Integers and Decimals.

From 1246.13 take 987.542.

Minuend,	1246.13	
Subtrahend,	987.542	
Remainder,	258.588	<i>Ans.</i>

As in addition, like units are placed in the same column. Assuming that the minuend is the sum of the subtrahend and the remainder, a

number is to be found that, added to 987.542, will give 1246.13. The

result may be obtained by placing under each figure of the subtrahend the figure which, combined with it, will produce the corresponding figure of the minuend, care being taken to provide for the carrying figures. Beginning at the left, say 2 and 8 (writing 8) are 10; carrying 1, 5 and 8 (writing 8) are 13; carrying 1, 6 and 5 (writing 5) are 11; carrying 1, 8 and 8 (writing 8) are 16; carrying 1, 9 and 5 (writing 5) are 14; carrying 1, 10 and 2 (writing 2) are 12.

While this method is more easily applied by young pupils, it is less frequently used in this country than the one next given.

567. Considering subtraction as taking one number from another,

$$\begin{array}{r} 11\ 13\ 15\ 10\ 12 \\ 1\ 8\ 5\ 0\ 2\ 10 \\ 1\ 2\ 4\ 6\ 1\ 3 \\ 9\ 8\ 7\ 5\ 4\ 2 \\ \hline 2\ 5\ 8\ 5\ 8\ 8 \end{array}$$

Ans.

the process may assume this form: Since 2 thousandths exceeds 0 thousandths, the 3 hundredths is reduced to 2 hundredths and 10 thousandths; 2 thousandths from 10 thousandths leaves 8 thousandths, written in the thousandths' column of the result.

Since 4 hundredths exceeds 2 hundredths, 1 tenth is reduced to 10 hundredths and added to the 2 hundredths, making 12 hundredths and leaving 0 tenths in the minuend; 4 hundredths from 12 hundredths leaves 8 hundredths. Since 5 tenths exceeds 0 tenths, the 6 units is reduced to 5 units and 10 tenths, etc.

In this example the line of small figures next above the minuend shows the successive reductions, the three in the hundredths' column being canceled and replaced by 2, and 10 thousandths being written in the proper column. When the 2 hundredths is increased to 12 hundredths, the 1 tenth is canceled and 0 written above it. This is next changed to 10 tenths, the 6 units being changed to 5, and so on.

NOTE. It is understood, of course, that the small figures illustrating the process should not be employed in doing the work.

568. Still another method is employed: a figure of the minuend

$$\begin{array}{r} 12\ 14\ 16\ 11\ 13\ 10 \\ 1\ 2\ 4\ 6\ 1\ 3 \\ 10\ 9\ 8\ 6\ 5 \\ 9\ 8\ 7\ 5\ 4\ 2 \\ \hline 2\ 5\ 8\ 5\ 8\ 8 \end{array}$$

Ans.

is increased by 10 units of the next order when necessary, and the next order in the subtrahend is increased by 1. This method, though more difficult to explain, is just as logical as the other, and is somewhat easier

in practice. It renders unnecessary a change in any figure of the minuend other than to prefix 1 to it when necessary, in which case the next figure of the subtrahend is increased by 1.

569. Oral Exercises.

- | | |
|-------------------------|----------------|
| 1. $641 - 300 - 80 - 7$ | 5. $742 - 387$ |
| 2. $932 - 400 - 60 - 5$ | 6. $824 - 465$ |
| 3. $820 - 500 - 70 - 3$ | 7. $940 - 573$ |
| 4. $764 - 200 - 90 - 5$ | 8. $653 - 295$ |

570. Subtraction of Mixed Numbers.

From $268\frac{1}{8}$ take $199\frac{19}{24}$.

	24ths	Make the fractions <i>similar</i> by reducing them to a common denominator. Since 3 is less than 19, the 8 units in the minuend is reduced to $7\frac{2}{3}\frac{4}{4}$. This added to the $\frac{3}{24}$ makes $7\frac{27}{24}$. From $\frac{27}{24}$ taking $\frac{19}{24}$ leaves $\frac{8}{24}$, which is written in the result, after being reduced to $\frac{1}{3}$. $267 - 199$ gives the integral part, 68.
	$268\frac{1}{8}$	
	$199\frac{19}{24}$	
<i>Ans.</i>	$68\frac{1}{3}$ $\frac{8}{24} = \frac{1}{3}$	

571. Subtraction of Compound Numbers.

From 10 oz. 7 pwt. 10 gr. Take 5 oz. 15 pwt. 20 gr. <i>Ans.</i> 4 oz. 11 pwt. 14 gr.	Since 10 gr. is less than 20 gr., it must be increased by 1 pwt., or 24 gr., making 34 gr. $34 \text{ gr.} - 20 \text{ gr.} = 14 \text{ gr.}$, written in the result. The 6 pwt. remaining is increased by 1 oz., or 20 pwt., making 26 pwt. $26 \text{ pwt.} - 15 \text{ pwt.} = 11 \text{ pwt.}$, written in the result. 5 oz. from the remaining 9 oz. leaves 4 oz., written in the result.
--	--

572. Written Exercises.

- | | |
|---|--|
| 1. $184\frac{3}{5} - 28\frac{7}{12}$ | 6. 16 rd. 3 yd. $- 4 \text{ rd. } 2 \text{ ft.}$ |
| 2. $16 - .00462$ | 7. $(7.5 - 3) - (.8 - .09)$ |
| 3. $280\frac{3}{10} - 146\frac{27}{1000}$ | 8. $21\frac{2}{3} - (16\frac{1}{5} - 3\frac{7}{15})$ |
| 4. 1 sq. yd. $- 15 \text{ sq. ft. } 94 \text{ sq. in.}$ | 9. $(4\frac{1}{2} - 1\frac{3}{5}) - (20\frac{1}{4} - 18\frac{3}{8})$ |
| 5. $3\frac{1}{2} \text{ A} - 475 \text{ sq. rd.}$ | 10. $1 - (.032 - .0075)$ |

573. Oral Problems.

1. The temperature yesterday was 18° above zero; to-day it is 3° below zero. How many degrees is to-day colder than yesterday?

2. Find the difference in latitude between a place in 60° north, and another in 35° south latitude.

3. If A is worth \$75 and B has nothing and also is in debt \$30, how much poorer is B than A?

4. M travels east 20 miles on a road, then 25 miles west on the same road, then 5 miles west, then 45 miles east; how many miles is he from his starting point? How far has he traveled?

MULTIPLICATION.

574. Multiplication is the process of obtaining a number called the **product** by performing, on a number called the **multiplicand**, the operation that must be performed upon unity to produce the **multiplier**.

575. Multiplication of Integers and Mixed Numbers. When the multiplier is an *integer*, the product is equal to the sum of the multiplicand used as an addend as many times as there are units in the multiplier. As the process of obtaining the product by addition is a long one, the following method is used.

1. Multiply 2468 by 3078.

$$\begin{array}{r}
 2468 \\
 \times 3078 \\
 \hline
 19744 \text{ units} \\
 17276 \text{ tens} \\
 7404 \text{ thousands} \\
 \hline
 7596504 \text{ Ans.}
 \end{array}$$

The product of 2468 by 8 units is first obtained. The product by 7 tens being tens, the first figure of this product is placed in the tens' column. The first figure of the product by 3 thousands is placed in the thousands' column. The sum of the partial products is the product required.

2. Multiply $567\frac{3}{4}$ by 27.

$$\begin{array}{r}
 567\frac{3}{4} \\
 \times 27 \\
 \hline
 4)81 \\
 20\frac{1}{4} \text{ units} \\
 3969 \text{ units} \\
 1134 \text{ tens} \\
 \hline
 15329\frac{1}{4} \text{ Ans.}
 \end{array}$$

27 times 3 fourths is 81 fourths, or $20\frac{1}{4}$, which is the first partial product. The product by 7 units is next written, the first figure being placed also in the column of units.

3. Multiply 567 by $27\frac{3}{4}$.

$$\begin{array}{r}
 567 \\
 \underline{27\frac{3}{4}} \\
 4) \overline{1701} \text{ product by 3} \\
 \quad 425\frac{1}{4} \text{ product by } \frac{3}{4} \\
 \quad 3969 \text{ product by 7} \\
 \quad 1134 \text{ product by 20} \\
 \hline
 \text{Ans. } 15734\frac{1}{4} \text{ product by } 27\frac{3}{4}
 \end{array}$$

Since 567 times 3 fourths has the same number of fourths as 3 times 567, this product is first written, and is then reduced to a mixed number by dividing by 4.

NOTE. When both the multiplier and the multiplicand are mixed numbers, their product is obtained by the method given below.

576. Multiplication of Fractions.

1. Multiply $\frac{4}{7}$ by $\frac{2}{5}$.

$\frac{4}{7} \times \frac{2}{5} = \frac{8}{35}$. *Ans.* $\frac{4}{7}$ multiplied by $\frac{2}{5}$ means $\frac{2}{5}$ of $\frac{4}{7}$. As 1 seventh means one of the seven equal parts into which a unit is divided, 1 fifth of 1 seventh means that this seventh is divided again into 5 parts, each of which is 1 thirty-fifth of the unit. Therefore 1 fifth of 4 sevenths is 4 thirty-fifths and 2 fifths of 4 sevenths is 2 times 4 thirty-fifths, which is 8 thirty-fifths.

2. Multiply $\frac{8}{9}$ by $\frac{3}{4}$.

$\frac{8}{9} \times \frac{3}{4} = \frac{2}{3}$. *Ans.* To multiply $\frac{8}{9}$ by $\frac{3}{4}$ is to find $\frac{3}{4}$ of $\frac{8}{9}$. 1 fourth of 8 ninths is 2 ninths, and 3 fourths of 8 ninths is 3 times 2 ninths, or 2 thirds.

3. Multiply $12\frac{3}{4}$ by $5\frac{1}{3}$.

$$\begin{array}{r}
 12\frac{3}{4} \\
 \underline{5\frac{1}{3}} \\
 4\frac{1}{4} = 12\frac{3}{4} \times \frac{1}{3} \\
 63\frac{3}{4} = 12\frac{3}{4} \times 5
 \end{array}$$
 Sometimes one mixed number may be multiplied by another in the manner shown in the accompanying example; as a rule, however, the mixed numbers should be reduced to improper fractions.

Ans. $68 = 12\frac{3}{4} \times 5\frac{1}{3}$ fractions.

$$12\frac{3}{4} \times 5\frac{1}{3} = \frac{51}{4} \times \frac{16}{3} = 68. \quad \text{Ans.}$$

577. To multiply one fraction by another, place the product of the numerators over the product of the denominators, canceling when possible.

578. Multiplication of Decimals.

Multiply .246 by .307.

$$\begin{array}{r} .246 \\ .307 \\ \hline 1722 \text{ millionths} \\ 738 \text{ ten-thousandths} \\ \hline .075522 \text{ Ans.} \end{array}$$

Written as common fractions, the process is as follows :

$$\frac{246}{1000} \times \frac{307}{1000} = \frac{75522}{1,000,000}.$$

The multiplicand and the multiplier each having three decimal places, the denominator is 1 with three ciphers in each case, and the product of the denominators will be 1 with three ciphers + three ciphers. To write this as a decimal requires six decimal places, or one place more than the numerator contains; the missing place is supplied by a decimal cipher.

579. To multiply one decimal by another, proceed as in the multiplication of whole numbers, and point off in the product as many decimal places as there are in the multiplier and the multiplicand together.

580. Finding a Per Cent of a Number.

What is 7% of \$347?

$$\begin{array}{r} \$347 \\ .07 \\ \hline \$24.29 \end{array}$$

To find 7% of \$347, multiply the latter by .07, 7% meaning 7 hundredths, or .07.

581. Multiplication of Compound Numbers.

Multiply £4 16s. 8d. by 7.

$$\begin{array}{r} £4 \ 16s. \ 8d. \\ \times 7 \\ \hline £33 \ 16s. \ 8d. \text{ Ans.} \end{array}$$

7 times 8d. = 56d. = 4s. 8d. Write 8d. and carry 4s. 7 times 16s. = 112s. Adding 4s. gives 116s. = £5 16s. Write 16s. and carry £5. 7 times £4 = £28. Adding £5 gives £33, which is written in the result.

582. The multiplicand may be concrete or abstract; the multiplier is always abstract.

When the multiplicand is concrete, the product is a like concrete number.

While we say, for instance, that the cost of 1874 articles at \$5 each is 1874 times \$5, in practice we employ 5 as the multiplier.

583. The multiplication sign, \times , is generally read "multiplied by," 7 lb. \times 5 = 35 lb., meaning 7 pounds multiplied by 5 equals 35 pounds. As the product of two numbers is the same, no matter what is the order of the factors, the foregoing is sometimes written 5 \times 7 lb. = 35 lb., in which case the sign must be read "times"; that is, 5 times 7 lb. = 35 lb.

NOTE. In expressions such as 3 ft. \times 7 ft., the sign \times should be read "by"; thus, 3 ft. by 7 ft.

584. Short Methods in Multiplication.

Pupils should have occasional drills in multiplying numbers without unnecessarily using their pencils. A few types are here given.

Sight Exercises.

- | | | |
|----------------------------|------------------------------|-------------------------------|
| 1. $24 \times .25$ | 5. 36×25 | 9. $64 \times .375$ |
| 2. $72 \times .125$ | 6. 48×125 | 10. 64×3.75 |
| 3. $33\frac{1}{3}\%$ of 66 | 7. $69 \times 33\frac{1}{3}$ | 11. 64×37.5 |
| 4. $12\frac{1}{2}\%$ of 88 | 8. $72 \times 12\frac{1}{2}$ | 12. $37\frac{1}{2} \times 64$ |

A number is multiplied by 99 by annexing two ciphers to it and deducting the original number; that is, 99 times 96 equals $9600 - 96$.

$88 \times 49 = (\frac{1}{2} \text{ of } 8800) - 88$. $48 \times 11\frac{1}{2} = (\frac{1}{8} \text{ of } 4800) - 48$. $27 \times 32\frac{1}{3} = (\frac{1}{3} \text{ of } 2700) - 27$.

- | | | |
|--------------------|-------------------------------|-------------------------------|
| 13. 99×99 | 16. $48 \times 11\frac{1}{2}$ | 19. $16 \times 61\frac{1}{2}$ |
| 14. 88×49 | 17. $66 \times 32\frac{1}{3}$ | 20. $16 \times 86\frac{1}{2}$ |
| 15. 36×24 | 18. $16 \times 36\frac{1}{2}$ | 21. $88 \times 99\frac{1}{2}$ |

The product of two numbers whose tens' figure is the same and the sum of whose units' figures is 10, may be obtained by prefixing to the product of the units' figures the product of the tens' figure by itself increased by 1. Thus, 84 times 86 may be written directly by prefixing 8 times $(8 + 1)$ to 4 times 6, 7224.

- | | | |
|--------------------|--|--|
| 22. 62×68 | 26. 5.5×5.5 | 30. 46×44 |
| 23. 74×76 | 27. $5\frac{1}{2} \times 5\frac{1}{2}$ | 31. 3.7×3.3 |
| 24. 83×87 | 28. 21×29 | 32. 95×95 |
| 25. 55×55 | 29. 2.1×2.9 | 33. $7\frac{1}{2} \times 7\frac{1}{2}$ |

To multiply a number by 100, 1000, etc., annex to the number as many ciphers as there are in the multiplier, or remove the decimal point a corresponding number of places to the right.

- | | | |
|------------------------|------------------------|------------------------|
| 34. 486×1000 | 37. 48.6×1000 | 40. 100×2.432 |
| 35. 243×2000 | 38. $.0486 \times 100$ | 41. 2000×2.43 |
| 36. 4.86×1000 | 39. 1000×3.75 | 42. $300 \times .123$ |

DIVISION.

585. **Division** is the process of obtaining a number called the **quotient** by performing, on a number called the **dividend**, the operation that must be performed upon the **divisor** to produce unity.

586. If horses are worth \$135 each, the cost of 27 horses is found by multiplication to be \$3645 ($\135×27). Two different problems in division may arise from this example in multiplication:

1. At \$135 each, how many horses can be bought for \$3645?
2. If 27 horses cost \$3645, what is the cost of one horse?

In the first example it is necessary to separate \$3645 into parts, each containing \$135; in the second example it is necessary to separate \$3645 into 27 parts.

$$\$3645 \div \$135 = 27, \text{ the number of horses.}$$

$$\$3645 \div 27 = \$135, \text{ the cost of one horse.}$$

Although different names are given to these different examples in division, the process is the same.

587. When the dividend and the divisor are like concrete numbers, the quotient is abstract. When the dividend is concrete and the divisor is abstract, the quotient is concrete, having the same unit as the dividend.

588. Divide 365,841 by 482.

$$\begin{array}{r}
 \text{Ans. } 759\frac{3}{482} \\
 482 \overline{)365841} \\
 \underline{3374} \\
 2844 \\
 \underline{2410} \\
 4341 \\
 \underline{4338} \\
 3
 \end{array}$$

Since the divisor is an integer, each partial quotient is of the same order of units as the partial dividend used to obtain it. The first partial dividend that will contain the divisor is 3658 hundreds, which contains the divisor 7 times. Place the 7 in the hundreds' place (just above the 8 of the partial dividend) and multiply the divisor by it, obtaining 3374 hundreds. Subtracting this product from the partial dividend gives a remainder of 284 hundreds, or 2840 tens. Bring down 4 tens from the dividend, making a new partial dividend of 2844 tens. Proceeding as before, the next quotient figure is 5 tens, and the new remainder is 434 tens, or 4340 units. Bring down 1 unit from the dividend, making a new partial dividend of 4341 units. The next quotient figure is 9 units, and the remainder is 3 units. The remainder is written as the numerator of a fraction, having the divisor 482 as the denominator.

589. Proof. The correctness of the result may be tested by adding the remainder to the product of the divisor and the quotient. If the sum is equal to the dividend, the work is correct.

590. Division of Decimals.

Divide .872355 by 5.8157.

$$\begin{array}{r}
 \text{Ans. } .15 \\
 58157 \overline{)8723.55} \\
 \underline{58157} \\
 290785 \\
 \underline{290785}
 \end{array}$$

Make the divisor a whole number by moving the decimal point four places to the right, and make a corresponding change in the dividend.

The number of places in the result will be equal to the number of places in the dividend.

NOTE. In the foregoing example the divisor and the dividend were multiplied by the same number, 10,000, which does not change the value of the quotient.

591. Division of Fractions.

1. Divide $117\frac{1}{2}$ by $\frac{3}{4}$.

$$\begin{array}{r}
 \frac{3}{4} \overline{)117\frac{1}{2}} \\
 \times 4 \quad \times 4 \\
 \hline
 3 \overline{)470} \\
 \underline{156\frac{2}{3}} \quad \text{Ans.}
 \end{array}$$

Multiplying the divisor and the dividend by the denominator of the fraction in the divisor gives a whole number for the divisor.

2. Divide $117\frac{1}{2}$ by $3\frac{3}{5}$.

$$117\frac{1}{2} \div 3\frac{3}{5} = \frac{235}{2} \div \frac{18}{5} = \frac{235}{2} \times \frac{5}{18} = \frac{1175}{6} = 32\frac{23}{6}. \quad \text{Ans.}$$

Change the dividend to the improper fraction $\frac{235}{2}$ and the divisor to $\frac{18}{5}$. To divide a number by $\frac{18}{5}$ the divisor is multiplied by 5, giving 18 as the new divisor, and the dividend is multiplied by 5. The quotient is 5 times the dividend divided by 18.

$$\begin{array}{r} 18 \overline{) \frac{235}{2} \times 5} \\ \underline{180} \\ 55 \\ \underline{54} \\ 1 \end{array}$$

or, $\frac{235}{2} \times \frac{5}{18}$.

An examination of the foregoing examples will show that in each case there has been performed on the dividend the operation that must be performed on the divisor in order to produce unity. In the first, the dividend has been multiplied by $\frac{4}{3}$; in the second by $\frac{5}{18}$. Each of these multipliers is the *reciprocal* of the corresponding divisor; that is, it is the divisor inverted.

592. To divide one fraction by another invert the divisor and proceed as in multiplication.

NOTE. Mixed numbers should be changed to improper fractions.

593. Divide $1872\frac{3}{5}$ by 11.

$11 \overline{) 1872\frac{3}{5}}$ When the divisor is an integer less than 13, the result $170\frac{13}{5}$ should be obtained by short division. Here the quotient is 170 with a remainder of $2\frac{3}{5}$, or $\frac{13}{5}$, one eleventh of which is $\frac{13}{55}$.

594. Compound Division.

1. Divide 18 gal. 1 pt. by 5.

Insert 0 qt. $18 \text{ gal.} \div 5 = 3 \text{ gal.}$, remainder 3 gal., or 12 qt. $12 \text{ qt.} \div 5 = 2 \text{ qt.}$, remainder 2 qt., or 4 pt. $4 \text{ pt.} + 1 \text{ pt.} = 5 \text{ pt.}$ $5 \text{ pt.} \div 5 = 1 \text{ pt.}$

$$\begin{array}{r} 5 \overline{) 18 \text{ gal. } 0 \text{ qt. } 1 \text{ pt.}} \\ \underline{3 \text{ gal. } 2 \text{ qt. } 1 \text{ pt.}} \end{array}$$

2. Divide 18 gal. 1 pt. by 3 gal. 2 qt. 1 pt.

$3 \text{ gal. } 2 \text{ qt. } 1 \text{ pt.} = 29 \text{ pt.}$ $18 \text{ gal. } 1 \text{ pt.} = 145 \text{ pt.}$

$29 \text{ pt.} \overline{) 145 \text{ pt.}}$ (5
 $\underline{145 \text{ pt.}}$

The dividend and the divisor must be of the same denominate unit. Reduce both to pints. $145 \text{ pt.} \div 29 \text{ pt.} = 5$.

595. Ratio.

Ratio is the relative value that exists between any two unequal like numbers. It is determined by division.

The ratio of 15 to 45 is $\frac{1}{3}$; that is, $15 \div 45$, or $\frac{1}{45}$ reduced to lowest terms. The ratio of 3 gal. 2 qt. 1 pt. to 18 gal. 1 pt. is the ratio of 29 pt. to 145 pt., or $\frac{29}{145}$, or $\frac{1}{5}$.

596. Only numbers of the same unit value can be compared.

597. Cancellation.

Operations involving only multiplication and division can be shortened by canceling factors common to the divisor and to the dividend.

598. Divide $40 \times 3 \times 1.89 \times 12$ by $6.3 \times 4 \times 72$.

$$\begin{array}{r} 5 \\ 10 \quad .3 \\ \hline 40 \times 3 \times 18.9 \times 12 \\ 63 \times 4 \times 72 \end{array} = 5 \times .3 = 1.5.$$

6
2

Write the factors composing the dividend above the line and those composing the divisor below, first changing 6.3 to a whole number and making a corresponding change in 1.89, the latter becoming 18.9.

Cancel 63 and 18.9, writing .3 over the latter. Cancel 12 and 72, writing 6 under the latter. Cancel 3 and 6, writing 2 under the latter. Cancel 4 and 40, writing 10 over the latter. Cancel 2 and 10, writing 5 over the latter. The remaining factors are 5 and .3 in the dividend. Their product, 1.5, is the result.

599. Multiplying or dividing the divisor and the dividend by the same number does not change the value of the quotient.

MISCELLANEOUS PROBLEMS.**600. Oral Problems.**

1. What is the ratio of $\frac{2}{3}$ to $\frac{4}{5}$ in whole numbers?
2. What part of a cubic foot is a board foot?
3. Into how many cubes $\frac{1}{4}$ ft. long can a cubic foot be cut?

4. One half of A's money equals one third of B's. B has \$1000 more than A. How much has each?

5. What is the date of maturity of a 90-day note drawn Sept. 7?

6. How many gallons of water must be mixed with 76 gallons of vinegar so that the mixture will contain 5% of water?

7. Find the cost at 1¢ per sq. ft. of painting a church spire whose base is a hexagon, each side measuring 10 feet, and the slant height being 60 feet.

8. A dealer bought 200 barrels of flour at \$5 per bbl. He sells 50 barrels that were damaged for \$4 per bbl., and the remaining 150 bbl. at \$6 per bbl. What per cent does he gain on the entire transaction?

9. A boy buys apples at 5 for 3¢ and sells them at 4 for 3¢. How many has he sold if his profit is \$3?

10. Three fourths of the value of A's house is equal to two thirds of the value of B's. What is the ratio of the value of A's house to that of B's?

11. What is the interest on \$600 at 4% for 270 days?

12. If the side of one square lot is 5 rods, and that of another is 10 rods, how do the lots compare in area?

13. Three fourths of M's farm is equal in value to two thirds of N's. Together the farms are worth \$17,000. What is the value of each?

14. A rectangular lot measuring 6 rods on one side has a diagonal of 10 rods. What is the area?

15. How many square rods in a field in the form of a trapezoid having parallel sides of 50 and 40 rods respectively and an altitude of 45 rods?

16. Three fourths per cent of A's money is \$60. How much money has he?

17. A man is engaged for a year for \$ 280 and a suit of clothes. At the end of 6 months he receives \$ 130 and the clothes. What is the value of the suit ?

18. Find the cost of 88 lbs. of coffee at $24\frac{7}{8}$ ¢ per lb.

19. If goods cost \$ 240 and the profit on them is 125%, what is the selling price ?

20. Find the value of a pile of wood 8 ft. by 4 ft. by 4 ft., at \$ 4 per cord.

21. A merchant buys goods at \$ 2 per yard ; he sells them 10% below the marked price and still makes a profit of $12\frac{1}{2}$ %. Find the selling price. The marked price.

22. In what time at 6% will \$ 100 amount to \$ 106.60 ?

23. A person bought stock at 20% above its face value and sold it at 10% below its face value. What per cent of the sum invested did he lose ?

24. What single discount is equal to successive discounts of 40 and 10% ?

25. After spending $\frac{2}{3}$ of his money and $\frac{1}{10}$ of the remainder a boy has 54 cents. How much had he at first ?

26. A man pays 60 cents for an article at a deduction of 40 and 10% from the list price. What was the list price ?

27. How many square yards of fencing 6 feet high will be required to build a tight board fence about a rectangular plot 150 feet long by 120 feet wide ?

28. In what time will a sum of money double itself at $4\frac{1}{2}$ % simple interest ?

29. Divide $1\frac{3}{8}$ by $\frac{3}{5}$.

30. How many acres are there in a field 20 rd. \times 32 rd. ?

31. A house is sold for \$ 4000, 25% of which is profit. What per cent would have been gained had the house been sold for \$ 3360 ?

32. What is the loss on 100 shares of stock bought at 90 and sold at 88, brokerage $\frac{1}{8}\%$ for buying and $\frac{1}{8}\%$ for selling?

33. What is the profit on 100 shares of stock bought at 88 and sold at 90, brokerage $\frac{1}{8}\%$ for buying and $\frac{1}{8}\%$ for selling?

34. How many gallons of ice cream will be needed to supply 128 people with $\frac{1}{2}$ pint each?

35. A rectangular field is 99 yards long and 87 yards wide. How many square yards does it contain?

36. What principal will produce \$ 150 interest in 2 yr. 6 mo. at 6%?

37. How many rectangular blocks 8 in. \times 4 in. \times 2 in. will be required to build a block 8 ft. \times 4 ft. \times 2 ft.?

38. Find the quotient of $302\frac{9}{10}$ divided by 10.

39. Multiply $20\frac{1}{2}$ by $6\frac{1}{2}$.

40. How many hours in April? In March? In February, 1908? In February, 1909?

41. A house costing \$ 4000 rents for \$ 30 per month. If the expenses for repairs, taxes, etc., are \$ 60 per year, what per cent does the owner make?

42. The ratios at which A, B, and C work are to each other as 2, 3, and 4. What whole numbers will represent, respectively, the time taken by each to do a piece of work?

43. Find the area of a plot of ground $9\frac{1}{2}$ yards long by $9\frac{1}{2}$ yards wide.

44. What sum invested in 6% bonds at 150 will yield \$ 600 per year?

45. Three fourths is what per cent of $\frac{2}{3}$?

46. By what per cent is the value of the fraction $\frac{3}{4}$ increased when 6 is added to the numerator and to the denominator?

47. At what rate of interest will \$100 in 3 yr. 4 mo. amount to \$115?

48. A woman spends $\frac{1}{5}$ of her money for a hat, $\frac{2}{3}$ of it for a dress, and has \$12 remaining. What had she at first?

49. At the rate of $37\frac{1}{2}$ cents a pint, how many gallons of olive oil can be purchased for \$15?

50. The volume of a cube is 1728 cubic inches; how many square inches in its entire surface?

51. Find the proceeds of a 60-day note for \$250, discounted the day it is drawn at 6%.

52. What number divided by $\frac{6}{7}$ gives a quotient of $\frac{2}{3}$?

53. How many pieces of ribbon each containing $\frac{7}{8}$ yd. can be cut from a piece containing 56 yd.?

54. Find the volume of a square pyramid 12 inches high, each side of the base measuring $5\frac{1}{2}$ inches.

55. I sold a carriage for \$240, on which I lost 20%; what did I lose? What fraction of \$240 did I lose?

56. What did I gain on a carriage sold for \$240, on which my profit was 20%? What fraction of \$240 did I gain?

57. Tea costing 20 cents a pound is mixed with an equal quantity of tea costing 30 cents a pound. What per cent is gained by selling the mixture at 35 cents per pound?

58. A can do a piece of work in 3 days and B can do it in 6 days. They work together and get \$12 for the work. How much should each receive? How many days do they take?

59. What will it cost to carpet a room 18 ft. long, 15 ft. wide at \$.75 per yard, the carpet being $\frac{3}{4}$ yd. wide?

60. A register 24 inches by 12 inches admits hot air to a room. Give the dimensions of a register that is double the size.

61. Give the inside dimensions of a box that will exactly contain 1 gallon (231 cubic inches).

62. Divide 8.4 by .4.

63. A quantity of sirup that has lost 20 % by leakage is sold by the gallon at 40 % above cost. What is the gain per cent ?

64. What is the cube root of 729 ?

65. Reduce $9\frac{7}{8}$ to an improper fraction, giving the reason for each successive step.

66. Multiply 66 by 64.

67. In how many years and months will any sum double itself at 6 % simple interest ?

68. Divide \$ 2100 among three persons so that the second may receive one half as much as the first, and the third one half as much as the second.

69. In the number 72,930, the local value expressed by the first two digits is how many times the local value expressed by the second digit ? By the third digit ? By the fourth digit ?

70. Find the cost of a two-inch plank 18 ft. long, 8 in. wide, at \$40 per M.

71. The capacity of a bin is 40 bu. of $11\frac{1}{4}$ cu. ft. What is the depth of the bin, if the length is 5 ft., and the width 4 ft. ?

72. How much coal at \$6 per ton is equal in value to 1800 lb. hay at \$12 per ton ?

73. A dealer paid \$950 for 200 barrels of flour. In addition he pays 20 ¢ per barrel for freight, and 5 ¢ per barrel for cartage. What must be the selling price of the flour per barrel, to produce a profit of 10 % on the total cost ?

601. Written Problems.

1. If a six-inch cube of stone weighs 21 pounds, what will be the weight of a cubic foot? How many times as heavy as water is the stone, a cubic foot of water weighing 1000 ounces?

2. Express in whole numbers the ratio of $\frac{25}{36}$ to $\frac{14}{27}$.

3. What rate of interest do I realize on stock paying 6% dividends, if it cost me 108, including brokerage?

4. Find the volume of a cube, whose entire surface contains 100.86 square inches.

5. I owe a bill amounting to \$311.85, for the payment of which I draw a 60-day note. What must be the face of a note which will yield the amount due, if discounted?

6. What is the diameter of a circular piece of ground that will contain 8 acres 106 square rods?

7. Add: 123,456; 78,695; 57,989; 6078; 85,769; 8888; 67,756; 200,009; 77,777; 85,685; 9999; 8,476,780; 57,869; 308,705; 96,878; 9586; 888; 57; 9.

8. What principal will produce \$1339.31 interest in 2 yr. 7 mo. 24 da. at 7%?

9. Sold $\frac{2}{3}$ of an article for what $\frac{3}{4}$ of it cost. What was the gain per cent?

10. What was the gain per cent in the sale of a horse when $\frac{4}{5}$ of the gain equaled $\frac{2}{3}$ of the cost?

11. How many cubic feet of water will be discharged in 5 hours from a pipe 2 feet in diameter, if the water flows at the rate of 10 miles per hour?

Volume of cylinder 2 feet in diameter, 50 miles high.

12. How many square feet in a plow furrow 1 mile long, $\frac{5}{6}$ foot wide? How many such furrows will contain an acre?

13. By selling goods at 46¢ per yard a loss of 8% is incurred. How much must this selling price be increased in order that a profit of 15% may be realized?

14. By what per cent is the value of the fraction $\frac{1\frac{3}{4}}{14}$ increased, when 6 is added to the numerator and to the denominator?

15. If 1800 sq. ft. of boards are needed to fence a lot 30 ft. by 120 ft., how many sq. ft. will be needed to fence a lot 25 ft. by 100 ft.?

16. What is the diagonal of a square field containing 20 acres?

17. It costs 8.25 francs to ship 3000 kilos 10 kilometers. What will it cost to ship 2400 kilos 62.5 kilometers?

18. A pile of 4-foot wood 36 ft. long is 12 ft. high at one end, and slopes regularly to a height of 8 ft. at the other; how many cords does it contain?

19. Divide \$150 between A and B so that A's share may be to B's share as $\frac{1}{2}$ to $\frac{1}{3}$.

20. If 3.5 yd. of cloth, 1.75 yd. wide, will make a suit of clothes, how much would be required if the cloth were only $\frac{3}{4}$ yd. wide?

21. The surface of a cube is 486 sq. in.; what is the surface of a cube one third the length of the first?

22. A cubical block of marble contains 27 cubic feet; how many cubic feet are there in a second cubical block having three times the dimensions of the first?

23. If 2 yd. 1 ft. 2 in. of ribbon cost 43 cents, what would have to be paid for 9 yd. 2 ft. 10 in.?

24. I gain 12% by selling silk for \$1.68 per yard. What should be my selling price to gain 24%?

25. A horse is fastened to a stake in the center of a field by a rope 75 feet long. If he can reach 2 feet beyond the rope, what fraction of an acre can he graze over?

26. A man is engaged for a year for \$300 and a suit of clothes. He receives the latter at once, and at the end of 7 months he leaves, receiving \$165 in cash. What is the value of the clothes?

27. What must be the face of a note that will yield proceeds of \$240, when discounted at a bank for 69 days at 6%?

28. A stick of timber is 16 inches broad and 10 inches thick; what length of it will make 10 cubic feet?

29. I invest \$19,950 in 4 per cents at 95, including brokerage. What annual income do they yield?

30. A floor 24 ft. 2 in. long and 18 ft. 11 in. wide is covered with carpet $\frac{3}{4}$ yd. wide, costing \$1 per yard. How much will be saved by running the strips the more economical way?

31. A merchant sold 20 hhds. of olive oil, each containing 63 gallons, at \$1.75 per gallon, and invested the proceeds in table sauce in cases of 12 bottles each, worth \$0.25 a bottle. How many cases did he buy? (Solve by cancellation.)

32. I bought a house and paid $\frac{1}{4}$ of the cost down, and $\frac{2}{3}$ of the remainder at the end of the year. The two payments amounted to \$9600. What was the cost of the house?

33. Multiply 2.4698 by 1000, and divide 64.2 by 10,000, by removing the decimal point in each case. Explain each operation.

34. If 7 bu. 2 pk. 7 qt. of chestnuts cost \$14.85, what are the chestnuts worth per bushel?

35. Which is the better discount on a purchase of dry goods amounting to \$1875.50, 40 and 10%, or 30 and 20%? What is the difference?

36. A gentleman wishes to endow a professorship in a college with an annual income of \$6000. What amount

must he invest for that purpose in 4% stocks at $122\frac{3}{4}$, brokerage $\frac{1}{8}\%$?

37. A debt which I incurred 5 yr. 3 mo. 15 da. ago, amounts, with interest at 7%, to \$2548.975. What was the original debt?

38. I own 65% of a mill and sell 40% of my share for \$5538. At that rate what is the mill worth?

39. A square prism 28 inches high contains 112 cu. in. How many inches square is it at either end?

40. A certain field is in the form of a trapezoid. Its parallel sides measure respectively 22.4 rods and 35.2 rods, and the perpendicular distance between them is 38.4 rods. What is the value of the field at \$160 an acre?

41. Show why removing a decimal point three places to the left divides by 1000.

42. Bought 3 hhd. 15 gal. 3 qt. catsup at the rate of \$42 per hogshead of 63 gal. If I bottle it in three-pint bottles costing 5 cents each, at how much a bottle must I sell it so that I may neither gain nor lose?

43. How many yards of paper 30 inches wide will be required to cover the walls of a room 15 ft. long, 11 ft. wide, and 7 ft. high, making no allowance for openings?

44. What part of $\frac{1}{12}$ of an acre is $\frac{9}{10}$ of a sq. rd.?

45. Find the sum of the following distances: 16 mi. 298 rd. 14 ft., 19 mi. 53 rd. 16 ft., 97 mi. 147 rd. 13 ft., and 47 mi. 237 rd. 13 ft.

46. A merchant sold a piece of cloth for \$33 and lost $8\frac{1}{3}$ per cent. What per cent would he have gained, had he sold it for \$43?

47. (a) At 5 per cent, find the present worth and the true discount of \$126.06, due in 8 mo. 12 da. (b) Regarding the foregoing problem as a problem in interest, tell

which of the terms used in interest are given, and which are required.

48. At what price must I buy stocks yielding an annual dividend of $4\frac{1}{2}$ per cent, that my investment may pay me 3 per cent interest?

49. If the services of eight men 8 hours a day for six days are worth \$132, what should be paid for the services of six men 10 hours a day for eight days?

50. A grocer mixed 8 lb. coffee worth 25 cents a pound with 12 lb. worth 30 cents a pound. At what price should he sell the mixture so that he may neither gain nor lose?

51. The large wheels of an engine are 16 feet 6 inches in circumference. How many revolutions will one of them make in traveling 5 miles?

52. A boy laid away 15 cents a day, Sundays included. What did his savings amount to from June 17, 1907, to May 12, 1908?

53. From a piece of land containing 75 A., I have sold ten lots each 30 rd. square. What is the remaining land worth at \$25.60 an acre?

54. A gentleman left $\frac{3}{4}$ of his estate to his wife, $\frac{2}{3}$ of the remainder to his son, and $\frac{3}{4}$ of what then remained to his daughter, who received \$750. What was the value of the whole estate?

55. What number must be multiplied by .085 that the product may be 1450.1? What number must be divided by .032 that the quotient may be 212.6 and the remainder .0008?

56. A note for \$325 was dated June 5, 1908. The rate of interest being $5\frac{1}{2}$ per cent, and no payment having been made on the note, how much was due at settlement, July 5, 1909?

57. Bought 87 shares in a mining company at 12 per cent below par, and sold the same at $19\frac{1}{2}$ per cent above par.

What sum did I gain, the par value of a share being \$75? No brokerage.

58. A bankrupt's assets are \$45,000, and his liabilities are \$67,500. How much can he pay on the dollar; and how much should A, to whom he owes \$12,675, receive?

59. Bought a hogshead of vinegar for \$21. Having lost $10\frac{1}{2}$ gallons by leakage, at what price per gallon must I sell the remainder to gain 25 per cent on the whole? (1 hogshead = 63 gallons.)

60. It is proposed to exchange a tank 9 ft. square and $1\frac{1}{8}$ ft. deep for another of equal capacity, but cubical in form. What will be the dimensions of the latter in feet and inches?

61. Find the value in pounds, shillings, and pence, of \$247.59, the pound sterling being \$4.8665.

62. New York is $74^{\circ} 3'$ west of London. When it is 4.30 A.M., solar time, at New York, what is the time at London?

63. A pound avoirdupois contains 7000 troy grains. At 80¢ per ounce troy, what is the value of a silver pitcher weighing 1 lb. 8 oz. avoirdupois?

64. A man bought 43.75 yards of carpet for \$58 $\frac{1}{3}$; he sold $\frac{2}{5}$ of it, gaining \$.16 $\frac{2}{3}$ on each yard sold. How much did he receive?

65. A merchant places a bill of \$840 in the hands of a collector, who succeeds in obtaining 75 per cent of it, and charges 5 per cent commission. How much does the merchant receive?

66. It is 80 rods between the diagonally opposite corners of a square field; how many acres in the field?

67. A grocer mixes 150 pounds of black tea with 80 pounds of green tea, and gains 38 per cent by selling the mixed tea for 30 cents per pound. If the black tea costs 20 cents a pound, what is the cost per pound of the green tea?

68. How much profit is made on 3600 meters of silk bought at \$1.60 per meter and sold at \$1.75 per yard? (A meter is 39.37 inches.)

69. A commission merchant sells for a farmer 45 tubs of butter containing 40 pounds each, for $18\frac{3}{4}$ cents per pound, charging him 3 per cent commission. With the proceeds he buys groceries, charging 3 per cent for buying. What is his commission on the sale, his commission on the purchase, and the cost of the groceries?

70. In building a concrete wall, 2 parts of lime, 1 part of cement, and 6 parts of broken stone are used. How many cubic feet of each are required in a wall $36' \times 9' \times 1\frac{1}{2}'$?

71. Solve the following question by cancellation. How many rolls of merino, each containing 75 yd., worth \$.45 a yd., will it take to pay for 180 yd. of alpaca at \$.30 a yd.?

72. Paid \$487 $\frac{1}{2}$ for a lot of apples, at the rate of \$.60 for $\frac{2}{5}$ of a barrel. How many barrels did I buy?

73. The difference in solar time between two places, *A* and *B*, is 5 hr. 5 min. 20 sec., *A* having the earlier time. If *B* is 18° west longitude, what is the longitude of *A*?

74. Sold certain goods for \$128.05, gaining \$9.85. What was the % of gain?

75. Find the face of a sight draft on St. Louis that may be bought for \$350, exchange being at \$1 premium per \$1000.

76. I have \$5000 in savings banks, drawing interest at 4%. If I invest it in 6% stocks at 125 including brokerage, how much more shall I receive annually than if I had kept it in the bank?

77. When may \$600 due in 4 mo., \$800 due in 5 mo., and \$1000 due in 10 mo., be paid at one time without loss to either debtor or creditor?

78. A cylindrical cistern contains, when full, 3080 cu. ft. of water. Its depth is 20 ft. What is its diameter?

79. The entire surface of a certain cube is 1014 sq. in. What is its volume?

80. Bought 140 yards of cloth at \$1.65 per yard. Sold $\frac{2}{5}$ of it at \$1.95 per yard. At what price per yard must the remainder be sold to gain 10 per cent on the whole?

81. The product of three factors is $1\frac{4}{5}$. One of the factors is $\frac{4}{5}$; the remaining two are equal. What are the equal factors?

82. What per cent do I gain or lose by deducting 20 per cent from the price of goods marked 20 per cent above cost?

83. In what time will \$1395 amount to \$1431.27 at 4 per cent?

84. Express in pounds and fraction of a pound the weight of a pint of water. (A gallon contains 231 cubic inches. A cubic foot of water weighs 1000 ounces.)

85. One side of a rectangular field containing $21\frac{3}{8}$ acres, measures 180 rods. What is the length of the diagonal of the field?

86. The population of a certain city was, in 1900, 14,500; in 1910, 124,250. What is the percentage of increase?

87. A man bequeathed one third of his property to his wife, one fourth of the remainder to each of three daughters, the rest to his son. The difference between his wife's share and that of his son was \$6750.75. How much did each receive?

88. Which is the greater discount, 40 and 20 per cent or 50 and 10 per cent? Why?

89. A bar of iron is 6 feet long, 15 inches wide, and 10 inches thick. How much is its length increased, when the width is reduced by rolling to 5 inches and the thickness to 4 inches, the bulk remaining the same?

90. A garden 25 ft. by 120 ft. is to be surrounded by a brick wall 5 ft. high and 12 inches thick. How many bricks will be required if 22 bricks with the mortar will lay one cubic foot?

Outside measurement of garden and wall, $27' \times 122'$; inside measurement, $25' \times 120'$. Difference = area of base of wall.

$$91. \left(2\frac{3}{8} \div \frac{1\frac{1}{2}}{3\frac{2}{5}} \right) + \left\{ \left(6\frac{5}{12} - 2\frac{3}{4} \right) \times \frac{2}{3} \right\} = ?$$

92. Write an analysis of the work required to solve the question: An officer in pursuit of a thief runs 8 yards to the thief's 5, but the latter has 60 yards start. How far does the officer run to overtake the thief?

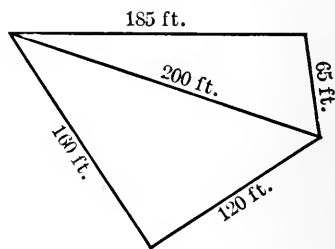
93. Divide twenty-five ten-thousandths by 125, multiply the quotient by five tenths, and divide 1000 by the product.

94. If 12 men can saw 45 cords of wood in 3 days, working 9 hours a day, how much can 4 men saw in 18 days, working 8 hours a day?

95. What per cent is gained or lost by selling $\frac{5}{8}$ of an article for the cost of $\frac{2}{3}$ of it?

96. Erastus Brooks owes me \$4489.32. He gives me his note, non-interest-bearing, at 90 days. The note discounted at once, at a bank, at 6 per cent, yields the exact amount of the debt. Required its face.

97. I once lent a friend \$875 for 1 year 4 months. He now proposes to lend me \$350 long enough to balance the obligation. What will the time be?



98. Find the area in square feet of the field represented by the following diagram:

99. A rectangular field whose breadth is $\frac{8}{15}$ of its length contains 3 acres. What is the length of the diagonal?

100. At 75 cents per yard, how much will it cost to carpet a room 19 ft. \times 15 ft. with carpeting $\frac{3}{4}$ yd. wide, the same figure recurring at intervals of 8 inches and the strips running lengthwise?

The length of a strip, being 19 ft., contains the pattern 28 times and 4 inches of the next repetition. To match at the top, the remaining 4 inches of this repetition must be cut off. The same waste occurs in each strip except the first.

101. Find the least common multiple of 35, 45, 63, 70.

102. Reduce 3.576 miles to its equivalent in miles, rods, and yards.

103. If coal is bought at \$6 per ton and sold at the rate of 30 cents per basket of 80 pounds, what is the gain per cent?

104. What single rate of discount is equal to successive discounts of 25, 20, and 5%?

105. Divide 64,564,000 by 798.

106. Reduce to its simplest form

$$\frac{\frac{5}{7} \text{ of } \frac{3}{10} + \frac{1}{4} \text{ of } \frac{8}{21}}{\frac{2}{3} \text{ of } \frac{9}{14} - \frac{5}{6} \text{ of } \frac{2}{15}}$$

107. The product of four factors is 432. Two of the factors are 3 and 4. The other two factors are equal. What are the equal factors?

108. What per cent is gained by selling at \$120 a thousand articles that cost \$9.50 a hundred?

109. A rectangular room measures 24 ft. by 18 ft. What will it cost to carpet it with material, three quarters of a yard wide, at \$1.12 $\frac{1}{2}$ a yard, if four yards are wasted in the matching?

110. How often may the quotient of 25 ten-thousandths \div 125 be subtracted from the quotient of $125 \div 25$ ten-thousandths?

111. A commission merchant in New Orleans whose charge for buying is $2\frac{1}{2}$ per cent, retained \$48 as his commission out of the money sent him for a purchase of sugar. What was the amount sent?

112. A man standing 150 feet from the foot of a tree 50 feet high, shot a bird hovering over the top; the man was 170 feet from the bird. How far was the bird from the top of the tree?

113. The exact interest (365 days to the year) of a certain sum of money at 4 per cent, from June 18 to August 30 (exact number of days), was \$40. What was the sum at interest?

114. Trees are set in an orchard at the intersections of lines drawn 2 rods apart. If the outside lines are each 1 rod from the boundaries, how many trees will be required in a 3-acre plot 24 rods long?

115. Find the exact interest on \$475 from July 16 to December 9 at 5%.

116. How many rods of fence will be needed to inclose a rectangular 40-acre field, one side of which measures $\frac{4}{5}$ mile?

117. What is the value, at \$4 per cord, of a pile of wood 16 feet long, 4 feet wide, and $9\frac{1}{2}$ feet high?

118. After paying an agent 5% of the sum collected, a man has \$1436.40. What was the sum collected by the agent?

119. A certain reservoir will hold 528,000 cubic feet of water. What must be the size of a square pipe which will fill the reservoir in 5 hours, if the water runs at the rate of 10 miles per hour?

The volume of water to flow hourly will be $\frac{1}{3}$ of 528,000 cu. ft. This may be considered as the volume of a square prism 5 miles high (26,400 ft.), each side of the base measuring x feet.

120. A wagon loaded with hay weighs 3175 lb. The wagon weighs 715 lb. What is the value of the hay at \$15.75 per ton?

121. How many bags, each containing 1 bu. 1 pk. 1 qt. 1 pt., will be required to hold 184 bu. 5 qt. of rye?

122. A meter is one ten-millionth part of a quarter of the earth's circumference. What is the length of the meter in inches, if the diameter of the earth is 7912 miles?

123. Give the reason for each step in the operation of finding the least common multiple of 3, 7, 9, 12, 14, 18.

124. How many pounds of bread can be made from 5 bu. wheat, weighing 60 lb. per bushel, if the wheat loses 30 per cent during the process of grinding into flour, and if the bread weighs $33\frac{1}{3}$ per cent more than the weight of the flour used?

125. A man loses \$310.50 on 360 barrels of flour by selling it at 15 per cent below cost. What is the selling price per barrel?

126. What principal will amount to \$715.13 in 2 mo. 13 da. at $4\frac{1}{2}$ per cent?

127. Two men start from the same place, one traveling north $4\frac{1}{2}$ miles per hour, the other going west at the rate of 6 miles per hour. In what time will they be 105 miles apart?

128. A man owns a city block 600 feet long, 240 feet wide. How many square yards of flagging will be needed to make a sidewalk 12 feet wide surrounding his property?

129. A's men can do a piece of work in 27 days, and B's men can do it in 36 days. In how many days will the work be done if one half of A's men and one third of B's men are employed?

130. A train going at a uniform rate starts from W at 7.15 A.M., and arrives at X at 9.45 A.M. After a stop of 15 minutes, it goes on to Y , where it also stops 15 minutes, and it reaches Z at 4.45 P.M. The distance between W and X is 100 miles, between X and Y 120 miles. Find the time of arrival at Y , and the distance between Y and Z .

131. What per cent is gained on beans bought at \$2.56 per bushel (2150.4 cu. in.), and sold at 33 cents per gallon, liquid measure (231 cu. in.)?

132. If a piece of wire 3 rods 2 yards 1 foot 6 inches long cost \$2.28, what will be the cost of a piece 19 rods 4 yards 2 feet 9 inches long?

133. A vessel has sailed due east $17^{\circ} 11' 15''$. Find the distance traveled in miles, the length of a degree being 48.64 miles.

134. In ten years the population of a state increased from 332,286 to 332,422. Find the percentage of increase. Find the percentage of increase in the population of another state in the same time from 98,268 to 328,808. Carry out to two places of decimals in each case.

135. A note for \$145.20 was discounted at a bank at 5 per cent, and the proceeds amounted to \$143.99. For how many days was the note discounted?

136. No allowance being made for mortar, how many bricks will be required to build a wall 3 rods long, 2 yards high, and 1 foot 3 inches thick, each brick being 8 inches long, 4 inches wide, and $2\frac{1}{2}$ inches thick?

137. At \$80 per acre, what will be the cost of a piece of land in the form of a right-angled triangle, the base measuring 120 rods and the perpendicular measuring 64 rods? Find the cost of fencing it at \$1.20 per rod.

138. At \$60 per M find the cost of 30 boards, each 16 ft. long, 16 inches wide.

139. Find the interest on \$1250 for 1 yr. 6 mo. 15 da. at $4\frac{1}{2}\%$.

140. A invests \$1200 and loses \$300 in 4 months; B invests \$750 and loses \$250 in 6 months. Which loses the greater per cent per month on his investment, and what per cent greater is his loss?

141. How much greater income fractionally will A's income be by buying 5% stock at 80 than by buying 7% stock at 117? (Brokerage included in prices given.)

142. Two men drive in the same direction around a square two miles on a side, starting from the same point at the same time. The first goes at the rate of five miles per hour and the second at the rate of six miles per hour. How far must the first go before they are again together?

143. How many gallons of vinegar are there in a mixture of $79\frac{1}{2}$ gallons containing 6% of water?

144. If the interest of \$625 for 3 yr. 7 mo. 6 da., at a certain rate per cent, is \$135, in what time will \$800 produce \$76 at $\frac{2}{3}$ as great a rate?

145. A cellar is 32 feet long, 8 feet deep, and 16 feet wide. What would be the depth of a cubical cistern of equal capacity?

146. A drover sold a lot of cattle at an advance of 20%. If they had cost him \$200, he would have lost 10%. What did he pay for the cattle?

147. A dealer mixed 45 bushels of oats, 37 bushels of corn, and 43 bushels of wheat for chicken feed, paying 25 cents for the oats, 50 cents for the corn, and 75 cents for the wheat per bushel. What is the cost of 5 bushels of the mixture?

148. Find the exact interest on \$3650 from April 14 to Sept. 6 at 5%.

149. Multiply 6472 by 9612.

150. What sum invested in 8 per cent bonds at $162\frac{3}{4}$ will yield an income of \$1200 per year?

151. At \$4 per cord what is the value of a pile of wood $18' \times 8' 6'' \times 4'$?

152. If a grain of gold can be beaten out into leaves covering 56 square inches, how many square feet will an ounce of gold cover when beaten out?

153. Multiply 9867 by $6097\frac{1}{2}$.

154. I sold a carriage for \$360 on which I lost 20%; what did I lose? I sold a carriage for \$360, on which I gained \$72; what per cent did I gain?

155. Find the interest on \$1080 for 1 year 6 months 15 days, at 5 per cent.

156. A farmer sells a pile of cordwood ten feet long, six feet high, and four feet wide at \$6 a cord. How many shingles at \$5 per thousand can he obtain for the money received through the sale of the wood?

157. What number is 325% of 1875?

158. State and solve a problem that will require the operations indicated below:

$$\frac{\$5.85 \times 150}{13 \times 18 \times 2\frac{1}{2}}$$

159. Divide 1.736 by 1.6; 1736 by .16; .1736 by 16.

160. A man pays \$324 for a piano. What is the "list price," if he has been allowed discounts of 40 and 10%?

161. A 90-day note for \$1200 was discounted at a bank June 27. The holder received \$1184, the rate being 6%. What was the date of the note?

162. What is the total surface of a cube, one edge of which measures $6\frac{1}{2}$ inches?

163. The divisor is 973, the quotient 8060, the remainder 549; what is the dividend?

164. Divide .63 by .4. Explain fully the reason for your location of the decimal point in the quotient.

165. How many half-pint bottles can be filled from a cask containing 24 gal. 3 qt. 1 pt. of vinegar?

166. What number multiplied by $2\frac{3}{8}$ and divided by $6\frac{1}{4}$ equals $3\frac{1}{3}$?

167. What is the sum of 7 miles 4 yards, 8 miles 225 rods, and 118 rods 3 yards? Give answer without fractions.

168. A and B own a mill, A's share being $\frac{5}{7}$ and B's the remainder. If A's share is worth £4381 2s. 6d., what is the value in English money of B's share?

169. A merchant buys green coffee for $16\frac{1}{2}\text{¢}$ per pound, and pays $\frac{1}{2}\text{¢}$ per pound for roasting. If the coffee loses 15% in weight during the roasting, what price must the merchant ask that he may gain 3 cents on each pound sold?

170. A man's expenses are \$1260, and he saves \$540; what per cent of his income does he spend?

171. If railroad stock sells at 160, including brokerage, what per cent semiannual dividends must be declared so that the stock may return 5% annually on the investment?

172. Find the entire surface of a square pyramid, whose altitude is 144 feet, one side of the base being 34 feet.

173. Find the weight of a cubic yard of granite, granite being 2.65 times as heavy as water, and a cubic foot of water weighing 1000 ounces.

174. What is the value of a carload of oats weighing 20,000 lb., at 42 cents per bushel of 32 pounds?

175. I owe \$900 on Oct. 16, and \$500 on Dec. 20. If I pay the former on Oct. 1, 15 days before it is due, at what date should I be permitted to pay the latter?

176. A farmer had 60 acres in oats, twice as much in wheat, and two thirds as much in corn as in wheat. His net profit per acre was \$6 on the wheat, which was 20% more

than his profit per acre on the corn, and 25% more than his profit per acre on the oats. How much more would he have gained if he had sowed all three parcels in wheat?

177. The Julian calendar assumed the length of a year as 365 da. 6 hr. instead of 365 da. 5 hr. 48 min. 49.7 sec. To how many hours did the difference amount in 100 years?

178. Four commercial travelers have routes which they cover in 30, 35, 40, and 45 days, respectively. If they start from headquarters the same day, how many days will elapse before all meet again at headquarters?

179. Divide \$1200 among three persons so that the second shall receive 25% more than the first, and the third 20% more than the second.

180. If a schoolroom 40 feet by 30 feet has 8 windows, each having 8 lights of glass 20 inches by 24 inches, what is the ratio of the lighting surface to the floor surface?

181. At 27¢ per cubic yard, find the cost of excavating a cellar in sloping ground, the length being 108 feet, the width 60 feet, the depth being 9 feet at one end, and $4\frac{1}{2}$ feet at the other.

182. The captain of a ship makes an observation of the sun at noon, at which moment his chronometer, keeping Greenwich time, indicates 2.15 P.M. In what longitude is the ship?

183. A fence 8 wires high is put around a square field containing 10 acres. Each strand of wire weighs 1 lb. per rod and costs $2\frac{1}{2}$ ¢ per lb., and the posts, placed 1 rod apart, cost 15 cents each. Find the cost of the fence.

184. At \$45 per M, find the cost of four sticks of timber each 16 ft. long, 12 inches wide, and 10 inches thick.

185. If 4¢ buys an 8-oz. loaf when flour is \$5 per barrel, how large a loaf should be bought for 6¢ when flour is \$6 per barrel?

TABLES

LINEAR MEASURE

12 inches (in.)	= 1 foot	ft.
3 feet	= 1 yard	yd.
5½ yards, or 16½ feet	= 1 rod	rd.
40 rods	= 1 furlong	fur.
320 rods	= 1 mile	mi.

$$1 \text{ mi.} = 320 \text{ rd.} = 1760 \text{ yd.} = 5280 \text{ ft.} = 63,360 \text{ in.}$$

A *hand*, used in measuring the height of horses, = 4 in. A *knot*, used in measuring distances at sea, = 1.15 mi. A *fathom*, used in measuring the depth of the sea, = 6 ft.

SQUARE MEASURE

144 square inches (sq. in.)	= 1 square foot	sq. ft.
9 square feet	= 1 square yard	sq. yd.
30¼ sq. yd., or 272¼ sq. ft.	= 1 square rod	sq. rd.
160 square rods	= 1 acre	A.
640 acres	= 1 square mile	sq. mi.

$$1 \text{ A.} = 160 \text{ sq. rd.} = 4840 \text{ sq. yd.} = 43,560 \text{ sq. ft.}$$

A **Section** of land is a square mile.

Roofing, flooring, and slating are often estimated by the *square*, which contains 100 square feet.

SURVEYORS' MEASURE

In measuring land, surveyors use a chain (ch.) which contains 100 links (l.) and is 4 rods long. Since the chain is 4 rods long, a square chain contains 16 sq. rd., and 10 sq. ch. = 160 sq. rd., or 1 acre.

CUBIC MEASURE

1728 cubic inches (cu. in.)	= 1 cubic foot	cu. ft.
27 cubic feet	= 1 cubic yard	cu. yd.
128 cubic feet	= 1 cord	cd.
16 cubic feet	= 1 cord ft.	cd. ft.
8 cord feet	= 1 cord	cd.

NOTE.—In computing the contents of an enclosing wall, masons and brick-layers regard it as one straight wall whose length is the distance around it on the outside. Corners are thus measured twice.

A *perch* of stone or masonry is 16½ ft. long, 1½ ft. thick, and 1 ft. high, and contains 24¼ cu. ft.

MEASURES OF CAPACITY

LIQUID MEASURE			DRY MEASURE		
4 gills	= 1 pint	. . . pt.	2 pints	= 1 quart	. . . qt.
2 pints	= 1 quart	. . . qt.	8 quarts	= 1 peck	. . . pk.
4 quarts	= 1 gallon	. . . gal.	4 pecks	= 1 bushel	. . . bu.

The *standard gallon* contains 231 cubic inches.

The *standard bushel* contains 2150.42 cubic inches.

The capacity of cisterns, reservoirs, etc., is often expressed in barrels (bbl.) of 31½ gallons each, or in hogsheads (hhd.) of 63 gallons each. In commerce, these vary in size.

AVOIRDUPOIS WEIGHT

16 ounces (oz.)	= 1 pound	lb.
100 pounds	= 1 hundredweight	cwt.
2000 pounds	= 1 ton	T.

One pound Avoirdupois = 7000 grains

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UNIVERSITY OF CALIFORNIA LIBRARY

8 drams	= 1 ounce	oz., or 3.
12 ounces	= 1 pound	lb., or lb.

One pound Apothecaries' weight = 5760 grains.

BRITISH OR STERLING MONEY

4 farthings	= 1 penny	d.
12 pence	= 1 shilling	s.
20 shillings	= 1 pound	£.
5 shillings	= 1 crown.		

The value of £1 is \$4.8665 in United States gold coin.

The unit of French money is 1 franc, which is 19.3 cents. The unit of German money is 1 mark, which is 23.85 cents.

